AN INVESTIGATION IN PRE-STRESSED CONCRETE

by

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AN INVESTIGATION IN PRE-STRESSED CONCRETE

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by

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The undersigned hereby certify that they have read and recommend to the School of Graduate Studies for acceptance, a thesis entitled "An Investigation in Pre-Stressed Concrete", submitted by Norman L. Reid, B.Sc., in partial fulfilment of the requirements for the degree of Master of Science.

PROFESSOR. I. F. Morrison
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Abstract

The development of pre-stressed concrete has received little attention on this continent until very recently. Most of the advancement in this subject can be attributed to English and European engineers. Such names as Freyssinet (France), Magnel (Belgium), Evans (England) are outstanding in this work.

Insofar as Western Canada is concerned no known projects of any consequence have been undertaken and little research has been carried out. In this investigation an attempt has been made to extend the general knowledge on the subject and perhaps create an interest in practical application. Specific attention was directed to the reliability of maintaining the tension in the reinforcing steel by bond alone. The results of the investigation indicate that for the form of reinforcing used (0.162 ins. diameter high tensile strength wire) mechanical anchorage devices must be provided in order to provide an acceptable factor of safety.

Acknowledgements

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Introduction

The purpose of this investigation is to extend the field of general knowledge in pre-stressed concrete, and to lay the foundations for further research in this subject to be carried out by the Department of Civil Engineering at the University of Alberta.

Specifically, the purpose is to reveal the nature of bond between concrete and the highly stressed reinforcing steel in a pre-stressed concrete beam. Moreover it is hoped to be able to determine whether bond alone is sufficient to maintain the tension in the reinforcing steel, or whether the specially designed (and usually patented) anchorage devices must be left in place.

It is intended to do this by testing two identical beams, one with the anchorages removed, the other with them retained, and to draw comparisons therefrom.

Observations pertaining to

(a) Effect of time on tension in the steel

(b) Load-deflection characteristics

(c) Behavior of the steel stresses under loading

(d) Occurrence and causes of failure

(e) Factors of safety

are to be recorded and reported upon.

The undertaking is also to contribute information of such practicable value as to promote the introduction of this type of construction to Western Canada.

Brief History of the Development of Pre-Stressed Concrete

The successful and comparatively recent applications of pre-stressed concrete, together with its current state of development, represents but one example of a desire on the part of engineers to use more efficiently and economically, the materials of construction. To this end therefore, the approach has been to exploit to the utmost their inherent strength characteristics.

It is common knowledge, of course, that plain concrete, while capable of withstanding large compressive stresses, can resist only very small tensile stresses in comparison. Thus it was that the use of concrete as a structural material was limited by its inherent weakness in tension. This limitation existed until designers realized the possibilities of reinforcing concrete with steel rods suitably placed to resist tensile stresses. This is made possible by the adhesion or bonding of concrete to the steel, thereby preventing any slipping of the rods. Hence this composite member acts as a single unit as it deforms under load. Carrying this process of thinking even further, the idea was postulated of imparting an initial compressive stress to composite ferro-concrete members. The fundamental purpose of pre-stress then, was to produce within the member an internal compressive stress which must be reduced to zero before any tensile stress can exist.

Pioneer attempts at pre-stressing -- Surprisingly enough, the idea of applying an initial tensile stress in the reinforcing steel was considered as early as 1886 by a German, C.F.W. Doehring. Patents were issued to him covering the manufacture of mortar slabs containing steel wire reinforcement which was tensioned before casting the mortar, and released after it had set up. He reasoned that this would produce a material whose components would fail more or less simultaneously and therefore increase the ultimate strength. Apparently he did not recognize the

fundamental purpose of pre-stressing -- viz, to produce an internal compressive stress that reduces or eliminates tensile stresses when the structure is loaded. However, this is not to be wondered at, since the theoretical principles and basic properties of ordinary reinforced concrete had still to be discovered.

It was in the period following that many early experiments in pre-stressed concrete were conducted and by about 1910 the basic principles appear to have been reasonably well established. In spite of the fact that the theoretical advantages of pre-stressing were then appreciated, the results of practical applications left much to be desired.

Failure of these pioneer attempts in arriving at a practicable system of pre-stressing may be attributed to two main reasons:

1. Inadequacy of the materials they had to work with. For example Doehrings pre-stressed slabs were unsuccessful because the quality of his cement mortar was so poor that the bond strength was insufficient to maintain the tension in the wires after they had been released.

Other German attempts failed because the permissible stress to which the steel could be tensioned at that time, was so low that the pre-stress disappeared because the elongation of the steel under load was smaller that the average shrinkage and contraction of the concrete.

2. Failure to appreciate the limitations inherent in the two materials -- this was particularly true in regard to <u>shrinkage</u> and <u>plastic behavior</u> of the concrete. (A good deal is still to be learned about this latter subject). In addition to this, the creep on the part of the steel tended to reduce if not eliminate the pre-stress.

<u>Development of Successful Methods</u> -- It was perhaps, E. Freyssinet who made the greatest single contribution towards making pre-stressed concrete " an economically and technically competitive structural engineering process".

It was he who discovered that the path to success was via the use of high quality concrete and high tensile strength steel. These are essentials.

High quality concrete provides the necessary bond strength and the desireable low shrinkage, low creep properties.

High tensile strength steel permits the use of tensile stresses of such magnitude that adequate pre-compression remains after losses due to shrinkage, contraction, creep, etc., have taken place.

Arrival at Practical Systems of Applying Pre-Stress -- The methods of application of pre-stress may be classified under one or other of two main systems.

(a) Pre-tensioning -- with this system the steel is placed under a tensile stress and anchored in this manner until the concrete has been placed and cured. After such time, the tensioning devices holding the steel are released thus transferring a compressive load to the concrete. This method may rely entirely on bond to hold the tension; or, specially devised grips may be placed on the ends of the steel to effect a mechanical anchorage.

(b) Post-tensioning -- this system arose as one of the early attempts to overcome the loss of pre-stress due to contraction of the concrete by shrinkage. This is accomplished by stressing the steel after all shrinkage has taken place.

In general, recent developments have tended to lie in the improvement of mechnical methods of which perhaps a few should be mentioned.

Notable in this regard is the double acting hydraulic jack invented and patented by E. Freyssinet. It is used in conjunction with a specially developed type of anchorage. The jack in tensioning the steel, finds its reaction on the face of the concrete, thus applying a compressive stress to the concrete as the wire is tensioned. This method is suitable

for post tensioning and is generally used to stress steel in a cable form.

In England, a method known as the "Lee-McCall" system makes use of high tensile silicon manganese steel bars with special threads and nuts at the ends "so designed as to develop practically the full strength of the bars without the necessity of upsetting". The procedure is to attach a special hydraulic jack to the bar by means of the threads and apply the desired load. This is measured by tapping the hydraulic system with a gauge. When this has been done the nut is tightened up against a previously provided bearing plate and the anchorage thus effected. A check on the stress in the bar can be made by measuring the strain.

One of the most interesting of the modern innovations is the device used in stressing wires around the circumference of a tank. Here a special machine has been adapted that travels in a spiral path around the circumference of the tank. The desired stress is obtained by passing the steel wire through a tight fitting circular orifice, the diameter of which must be pre-determined according to the design stress.

This by no means exhausts the discussion on the systems of applying pre-stress but serves to illustrate some of the work in that regard. Coincident with the development of stressing devices must go the inventions of various anchorages such as the Freyssinet cones, Magnel sandwich plates, Gladwin plugs, etc.

Advantages of Pre-stressed Concrete -- With the foregoing contributions in mind, then, it is felt that the specific advantages of pre-stressed concrete which motivated this intensive study, should be set down.

Paramount, of course, pre-stressing compensates for the missing property of concrete -- namely tensile strength. In reality it does more -- for it enables us to use the small tensile strength that concrete does have. This then makes for economical use of both steel and concrete because

it permits the designer to work his steel at stresses near ultimate, whereas, formerly this was drastically restricted in order to prevent cracking. Additionally the compressive strength of the concrete may be fully exploited.

A very desireable advantage that is apparently overlooked in most literature, is the reduction in diagonal tension. In ordinary reinforced concrete where the shearing forces are high, an uneconomically large beam is necessitated. Pre-stressed concrete permits the use of smaller units and in some cases eliminates the need for stirrups.

In application to bridge design, for example, this means savings in dead load can be effected that could never be approached with ordinary reinforced concrete. Associated directly with this is the added "head room" that becomes available due to the reduced girder size possible. Reference here is made to the pedestrian bridge now being constructed in Los Angeles under the direction of L.C. Hollister. Comparative designs show a gain of 3'4" head room for a pre-stressed structure as compared with ordinary reinforced concrete. The span is 110 feet, with an 8'0" clear walk. Savings in materials were computed to be 38 cu. yd. for concrete, and 33,000# of reinforcing steel.

Finally, where occasional overload may cause excessive deflection accompanied by inevitable cracking, the effect of pre-stress, like a "benevolent guardian" is to return the member, on removal of the load, to an undeflected position, whereupon the cracks are complete closed making their detection by eye almost impossible.

It would be wrong to have conveyed the impression that the application of pre-stressed concrete is limited to beam and girder type construction. In reality, the scope is wide and varied, for use has been

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made of this principle in the design of tanks, reservoirs, railway ties, barges, transmission line poles, tunnels, anchorage for additional superstructure to increase dam heights, and even an experimental ship of about 500 tons.

Having thus traced, very briefly, the steps in the development of this "new" method of construction, the author feels that one thing becomes rather significant -- that pre-stressed concrete is <u>not a competitor</u> to ordinary reinforced concrete, but, the natural and evolutionary step in the advancement of what is one of our most important structural materials.

<u>Outline of the Investigation</u>

Firstly, it must be realized that the term bond, means nothing more than the anchorage, without resort to mechanical devices, of steel to mortar. It is made up of two components. One is the straight adhesion between the steel and the surrounding mortar. The second is a frictional or slippage resistance. The latter effect is incipient in nature, that is it is not available for use until the first component has been overcome and the steel starts to move within the concrete. It is the larger of the two components, but unfortunately little advantage can be taken of it in pre-stressed concrete. This follows, since the very movement by virtue of which it arises is generally sufficient to eliminate the tension in the steel upon which the principle of pre-stressed concrete depends.

This, however, suggests a means whereby bond may be investigated. The method, obviously, is to measure the stress in the reinforcing steel at periods subsequent to pre-stressing and under varying conditions of load for a case where bond alone is anchoring the steel. The degree to which this stress is retained is therefore a measure of the extent to which bond

is responsible for effecting anchorage.

Testing was carried out on two pre-stressed concrete beams, made in the laboratory, specifically for that purpose. A 10 foot span with a 6" x 10" rectangular cross-section was used in each case. Reinforcing steel consisted of 24 separate strands of 0.162" diameter high tensile strength wire, stressed to approximately 130,000 p.s.i. Concrete was of high quality, approximately 5000 p.s.i. based on standard 6" x 12" cylinders.

Four of the wires were selected for special study and accordingly wire-resistance strain gauges were attached at strategic locations along their length.

After curing for 28 days, the beams were subjected to cyclic loading in a Riehle Testing Machine. Beam #1 was tested after having the mechanical anchorage devices removed, while Beam #2 underwent testing with the anchorage left in place. Subsequent to the analysis and interpretation of the data recorded during the tests, the following conclusions were drawn:

1. Anchorage by bond alone did not give what is considered to be an adequate safety factor. It was found to be only 1.59 (Beam #1) as compared to 2.82 for the case where the anchors were retained (Beam #2).

2. Beam #1 failed due to loss of bond, that is a slippage of the wires. Beam #2 failed due to a compression failure in the concrete.

3. Recovery characteristics over the design load for Beam #1 were poor while the opposite was true for Beam #2.

4. Pre-stressing precluded the possibility of failure by diagonal tension and obviated the necessity of stirrups.

5. Indications are that the transmission length required for 0.162 inch diameter wire stressed to approximately 130,000 p.s.i. is of the order of 108 times the diameter.

DESIGN OF THE TEST BEAMS

A. Notation

A, cross-sectional area of beam.

b, breadth of rectangular beam or of flange of tee-beam.

C, C_1, constants relating to the ratio of stresses.

c, permissible compressive stress in the concrete.

c_{ab}, c_{at}, calculated stresses in the bottom and top fibres respectively due to w_a.

c_{db}, c_{dt}, calculated stresses in the bottom and top fibres, due to loads acting at the time of prestressing.

c_t, permissible tensile stress in the concrete.

D, total depth of beam or slab.

d, effective depth of beam or slab.

E_c, E_s, elastic modulus for concrete and steel respectively.

I, I_v, moments of inertia about the horizontal centroidal axis and about the vertical axis of symmetry respectively.

M, total bending moment.

M_a, M'_a, bending moments due to w_a acting with and contrary to M_d respectively.

M_d, bending moment due to w_d.

P_i, initial stretching force.

Q, Q_1, factors for moment of resistance for ordinary reinforced concrete and for prestressed concrete respectively.

Q_m, moment about the neutral axis of the area of a section on one side of the neutral axis.

R, vertical component of force in an inclined cable.

r, radius of gyration of the concrete section.

S, shearing force. (S_{EA}, etc., shear force at EA, etc.)

t, permissible tensile stress in the steel.

w_a, additional load per unit length applied after the prestress has been established.

w_d, load per unit length acting when the prestress is being established.

y_1, y_2, distances from the centroid to the top and bottom fibres respectively; for a symmetrical section $y_1 = y_2 = y$.

e, eccentricity of the stretched wires from the centroid.

e_A, e_B, eccentricity applicable to points A and B respectively.

η, proportion of P_i that remains permanently; generally $\eta \doteq 0.85$.

B. Design Procedure

The out to out length was chosen as 11 feet, in accordance with the existing laboratory testing apparatus which permitted the use of spans up to approximately 10 feet center to center of supports.

A simple rectangular beam was decided upon as a matter of ease in forming. This was permissible since the purpose of the test was not to investigate any particular cross-sectional shapes. The depth was taken as twice the width. This is roughly in agreement with economical design practice.

Data

$c = 2000\#/in^2$

$c_t = 0$

$t = 150{,}000\#/in^2$

$\eta = 0.85$

$D = 12$ in.

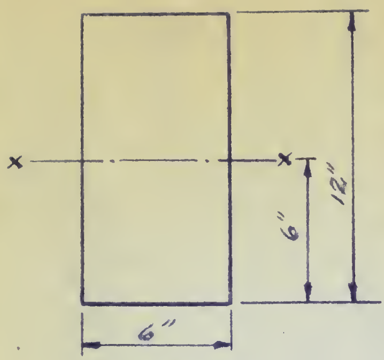
$b = 6$ in.

$\ell = 10'$-$0"$

wire dia. $\doteq$ 0.167 in.

wire cross-section area = 0.0206 sq.in.

For derivation of the equations that follow, refer to Appendix "A".



Section

Fig. 1

Section Properties

$$A = 6 \times 12 = 72 \text{ in}^2$$

$$I = \frac{6 \times 12^3}{12} = 864 \text{ in}^4$$

$$r^2 = \frac{864}{72} = 12 \text{ in}^2$$

$$r = 3.464 \text{ in.}$$

$$S_m = \frac{864}{6} = 144 \text{ in}^3$$

Note, that here the problem is reversed to what is generally encountered in that the dimensions of the beam have been initially decided upon and the allowable load is to be computed. In practice, of course, the size of the beam is unknown and must be arrived at on the basis of the loading conditions.

Considering a section at mid-span -

By Eqn.(12) $$Q_1 = \frac{0.775c + c_t}{6}$$

$$= \frac{0.775 \times 2000 + 0}{6}$$

$$= 258.3$$

By Eqn.(11) $Q_1 \, b \, D^2 \geq M_a$

Thus for full design load

$M_a = 258.3 \times 6 \times 144 = 223{,}200'\#$ which is the allowable bending moment due to the live load. This load is to be applied by the testing apparatus as two concentrated loads P at the third points of the span.

$$P = \frac{223{,}200}{10/3 \times 12} = 5{,}580\#$$

Assuming concrete to weigh 150#/cu.ft. the dead load

$$w_d = \frac{72 \times 150}{144} = 75\#/\text{ft.}$$

Therefore dead load moment

$$M_d = \frac{75 \times 10^2}{8} \times 12 = 11{,}250''\#$$

See Plate I for shearing force and bending moment diagrams due to these loads.

The maximum compressive stresses in the concrete due to M_d and M_a respectively, are:

$$c_{dt} = c_{db} = \frac{11{,}250 \times 6}{864} = 78.1\#/\text{in}^2$$

$$c_{at} = c_{ab} = \frac{223{,}200 \times 6}{864} = 1550.0\#/\text{in}^2$$

$$\overline{1628\#/\text{in}^2}$$

Thus $c > c_{dt} + c_{at}$ since $2000 > 1628$

Hence from Eqn.(18)

$$C = \frac{(c - c_{dt} - c_{at})}{c_{db} + c_{ab} - c_t}$$

$$= \frac{0.85\ (2000-1628)}{1628 - 0}$$

$$= .1942$$

and from Eqn.(19)

$$e_A = \frac{(1-c)\ r^2}{y_1 + cy_2}$$

$$= \frac{(1 - .1942)\ 12}{(6 + .1942 \times 6)}$$

$$= 1.35 \text{ in.}$$

This eccentricity can easily be obtained with the dimensions previously assumed and no further refinement need be made in the depth.

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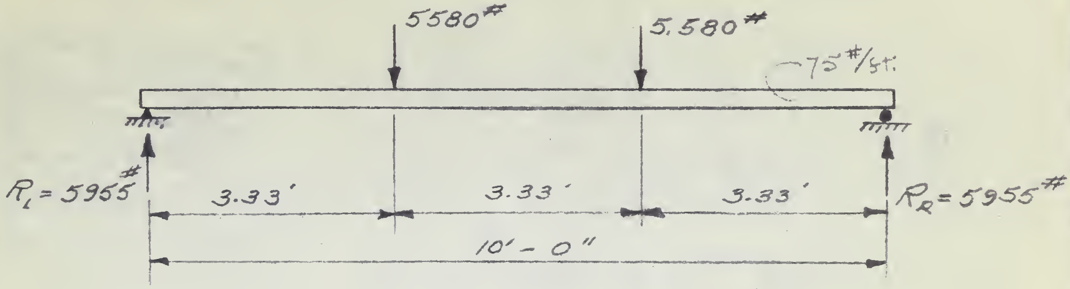
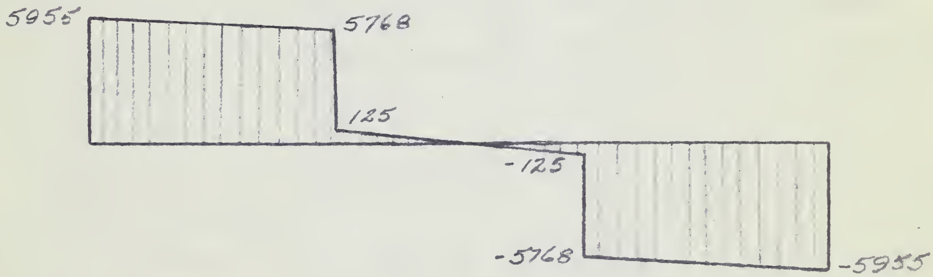
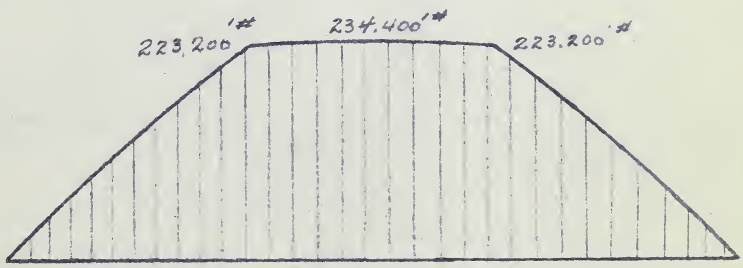




Shearing Force Diagram



Bending Moment Diagram

PLATE I
BEAM DESIGN

Computations for the four lines composing the graphical representation of the basic conditions which must be satisfied, now follow:

For line (1): $$-\frac{1}{(c_{dt} + c_t)A} = -\frac{1}{(78.1 + 0)A} = -\frac{12.8}{1000\ A}$$

For line (2): $$-\frac{\eta}{(c_{dt} + c_{at} - c)A} = -\frac{0.85}{(1628 - 2000)A} = +\frac{2.28}{1000\ A}$$

For line (3): $$\frac{1}{(c + c_{db})A} = \frac{1}{(2000 + 78.1)A} = +\frac{.481}{1000\ A}$$

For line (4): $$\frac{\eta}{(c_{db} + c_{ab} - c_t)A} = \frac{0.85}{(1628 - 0)A} = +\frac{.522}{1000\ A}$$

These are shown plotted on Plate II - Graph of Basic Conditions.

The maximum value of eccentricity obtained from the graph is 2.2 in. giving a corresponding value of

$$\frac{1}{P_i}(1000\ A) = 1.1$$

$$P_i = \frac{1000 \times 72}{1.1} = 65{,}500\#$$

These values may be computed agebraically and with more accuracy in this particular case. The common solution to the equations for lines (1) and (4) gives the desired co-ordinates.

For line (1): $$\frac{1}{P_i} \geq \frac{\frac{e\ y_1}{r^2} - 1}{(c_{dt} + c_t)\ A}$$

For line (2):

$$\frac{1}{P_i} \leq \frac{(1 + \frac{e\ y_2}{r^2})}{(c_{db} + c_{ab} - c_t)\ A}$$

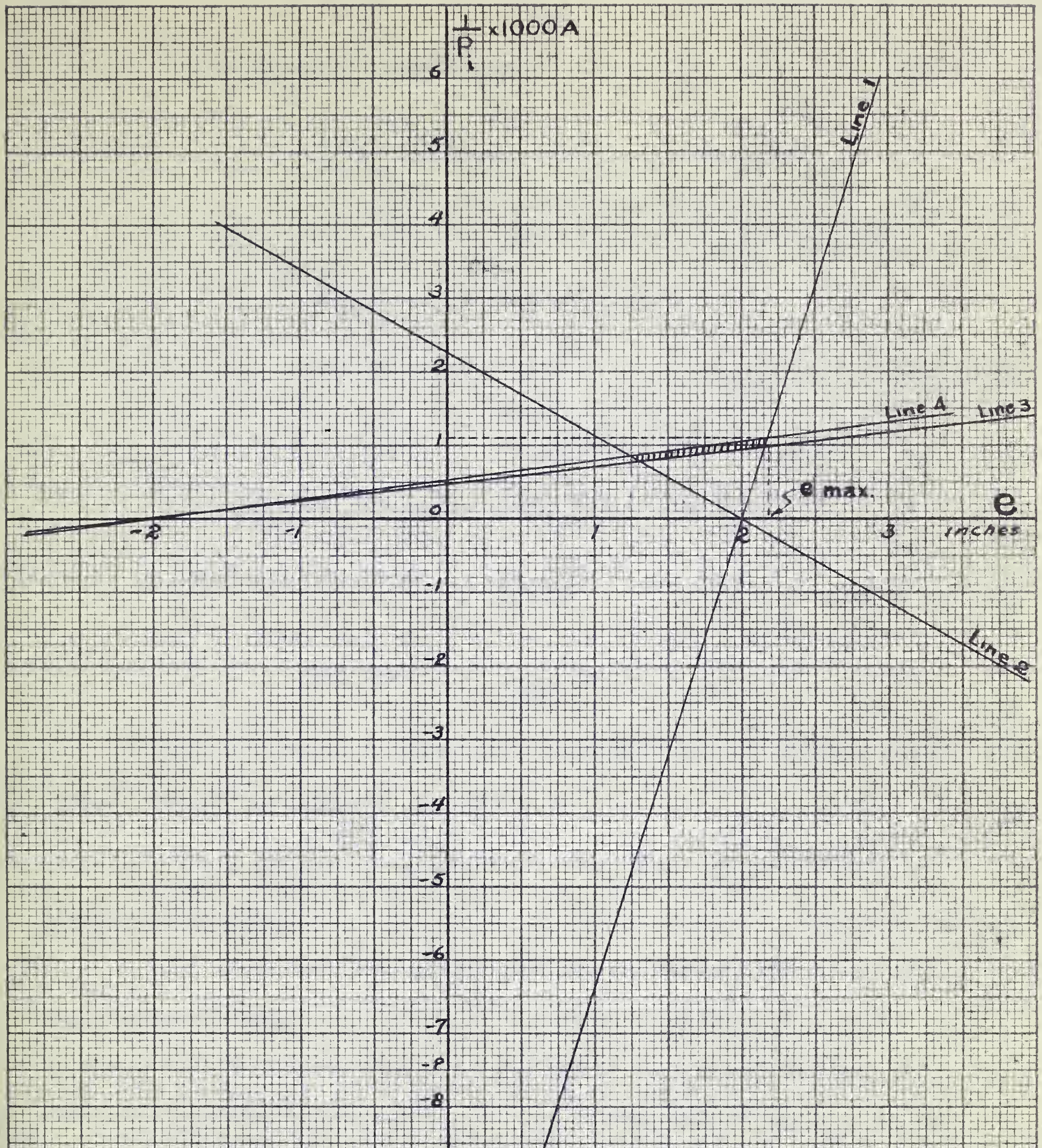


PLATE II

BEAM DESIGN

Graph of Basic Conditions

At the point of intersection

$$\frac{\frac{e\,y_1}{r^2} - 1}{(c_{dt} + c_t)\,A} = \frac{\eta\left(1 + \frac{e\,y_2}{r^2}\right)}{(c_{db} + c_{ab} - c_t)\,A}$$

$$\frac{.5\,e - 1}{78.1} = \frac{0.85\,(1 + .5\,e)}{1628}$$

$$\text{solving,} \quad e = 2.170 \text{ in.}$$

$$\frac{I}{P_i} = \frac{\frac{2.170}{2} - 1}{(78.1)\ 72}$$

$$P_i = \frac{72 \times 78.1}{.0850} = 66{,}155\#$$

$$\text{Area steel required} = \frac{66{,}200}{150{,}000} \quad .442 \text{ sq.in.}$$

$$\text{No. wires} = \frac{.442}{.0206} = 22$$

Use 24 wires in 6 layers of 4 wires per layer. Therefore required load in each wire $= \frac{66{,}200}{24} = 2760\#$.

Arrangement of Wires

Any convenient arrangement of wires whatsoever, resulting in the required eccentricity, may be used. It must be borne in mind, however, that this eccentricity was computed for a section at mid-span of the beam, or section of maximum bending moment. At the extreme ends of the beam it is found desirable to reduce the eccentricity to zero, for this may be accomplished in such a manner so as to materially reduce the shearing stress and also preclude the possibility of a tensile stress in the top fibre when dead load only is acting.

The point at which reduction in eccentricity should commence and its effect on shear is dealt with in detail in the next few pages.

The system of wires shown in Figure 2 was decided upon.

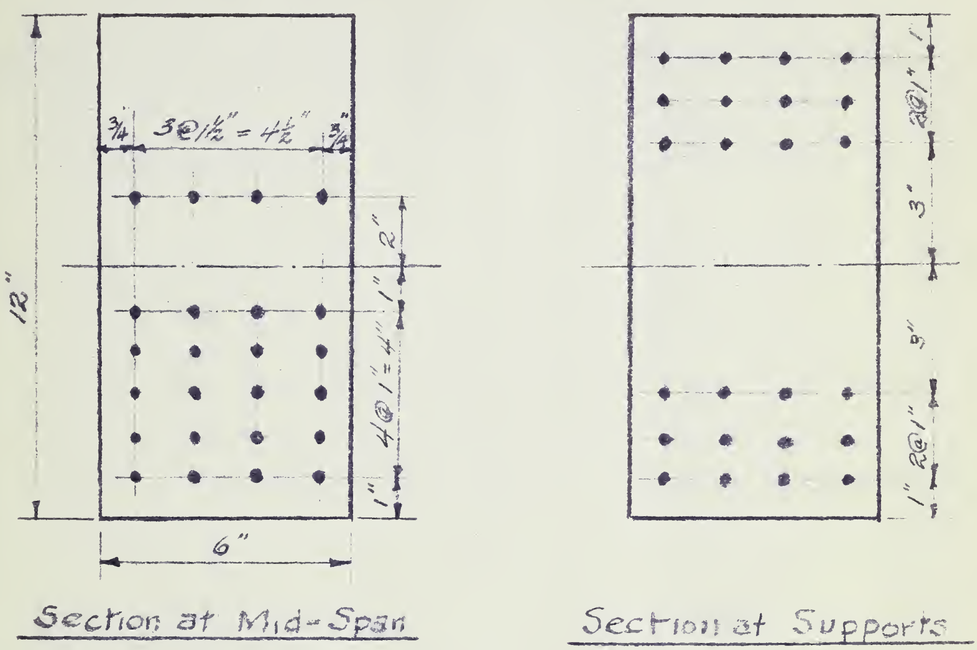


Fig. 2

A simple check (by summing moments of the layers of wires about any axis) shows that the eccentricity "e" is the value required by previous design.

$$6e = 1 + 2 + 3 + 4 + 5 - 2 = 13.0$$

$$e = \frac{13.0}{6} = 2.17 \text{ in.}$$

Reduction of Eccentricity

As sections are considered more and more remote from the mid-point of the span, the bending moment arising from dead and live loads decreases and becomes zero over either support. Accordingly then it may be essential to reduce the amount of pre-stress in this region in order that excessive tensile stresses are avoided in the top fibre when the dead load alone is acting. The particular design at hand does not illustrate this point exceedingly well because the dead load is comparatively small. This is, nevertheless, an extremely important factor in cases such as bridge girders where dead loads are appreciable and spans are abnormally great.

In any case, the reduction of eccentricity may be utilized to aid in reducing the shearing stress. The reduction of eccentricity is best accomplished by raising some of the top layers of reinforcing steel near the ends of the span. Generally this is carried out such that the eccentricity becomes zero over the support. The selection of the point at which this reduction should be started is best illustrated by means of a graph of the stresses in the extreme fibres. (See Plate III). The various lines indicate the different components which go together to make up the net fibre stress. (Note that compression and tension, although of opposite sign, are plotted on the same side of the datum to facilitate indicating the net stress). The values needed to plot such a graph are given in the following tables.

Bottom Fibre

	Section At					
	Mid Span		1/3 Pt.		End Pt.	
Component	Initially	Ultimately	Initially	Ultimately	Initially	Ultimately
Dead Load	+ 78.1	No change	+ 69.5	No change	0	0
Live Load	+1550	No change	+1550	No change	0	0
P_i/A	- 918	- 780	- 918	- 780	- 918	- 780
$\frac{P_i e\ y_2}{I}$	- 996	- 847				

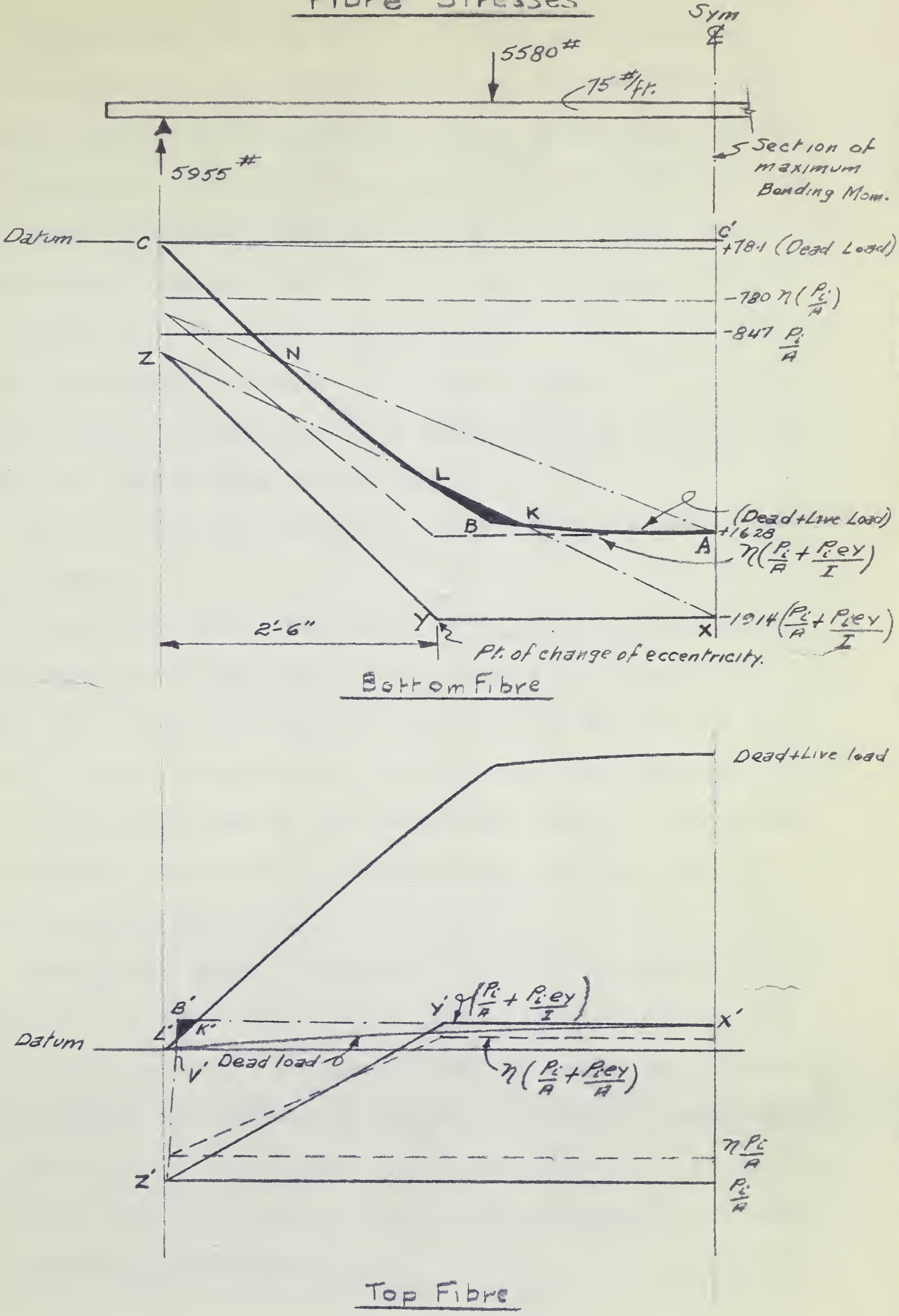
Top Fibre

	Section At					
	Mid Span		1/3 Pt.		End Pt.	
Component	Initially	Ultimately	Initially	Ultimately	Initially	Ultimately
Dead Load	- 78.1	No change	- 69.5	No change	0	0
Live Load	-1550	No change	-1550	No change	0	0
P_i/A	- 918	- 780	- 918	- 780	- 918	- 780
$\frac{P_i e\ y_1}{I}$	+ 996	+ 847				

The values of $\frac{P_i e\ y}{I}$ for sections at the 1/3 and End points have been left blank, since presumably the eccentricity would not, at this stage, have been decided upon for those regions.

It should be noted that in preparing the graphs for Plate III the values in the above tables were not plotted to scale, but certain conditions were exaggerated to illustrate the point in question.

Fibre Stresses



(not to scale)

PLATE III
BEAM DESIGN

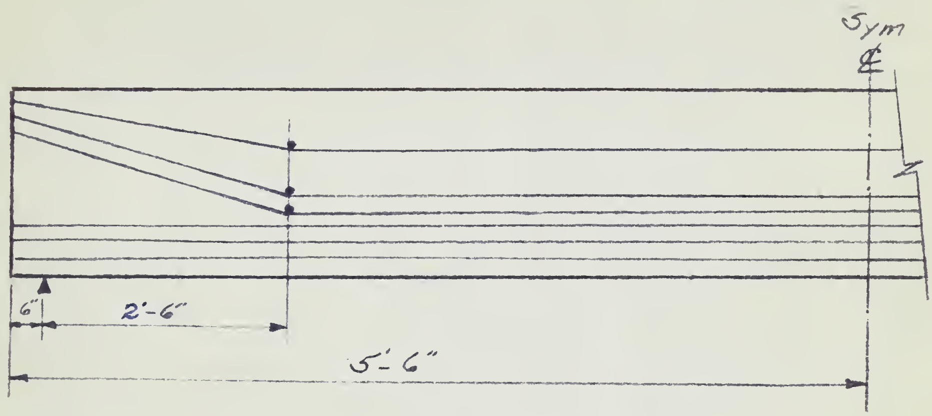
With reference to Plate III it is noticed that the point of commencement of reduction in eccentricity has been chosen at 2'-6" from the support. Considering the bottom fibre this is permissible, since the line XYZ representing fibre stress resulting from pre-stress for this case, does not cut the line ABC of fibre stress caused by the loading terms. The reduction could not have been started at mid-span, for example, since the line X-Z representing this case cuts the line ABC at K and L, thus indicating a tensile fibre stress. Furthermore this tensile stress would obviously be increased in value and broadened in scope from area BLK to area BNA as the anticipated loss of pre-stress materializes.

Similarly any other choice which permits a tensile fibre stress should be discarded.

On the other hand, consideration of the graph of stress in the top fibre shows that the limit has not been exceeded in the opposite direction. That is, had the reduction of eccentricity been started at a point too far distant from mid span, a tensile stress would have developed in the top fibre -- for example, the line X'-B'-Z' results in the tensile stress indicated by area K'B'L' when dead and live loads act, and area X'B'V' when dead load only acts.

For the case chosen, indicated by line X'Y'Z', it should be mentioned that a slight tensile stress in the top fibre is possible when dead load alone is acting. Note, however, that the maximum value (occurring at the section at Y') is only 19.5 p.s.i. This is thought to be permissible, not only because of the small magnitude, but also because, that as pre-stress decreases with time, this tensile stress will also decrease and eventually will be replaced by a compressive stress.

The actual reduction of eccentricity was effected by placing dowels through the form walls (see Figure 3) so that the top three layers of steel could be bent upward in such a manner that the eccentricity was reduced linearly to zero at the end of the beam. (See Figure 3)



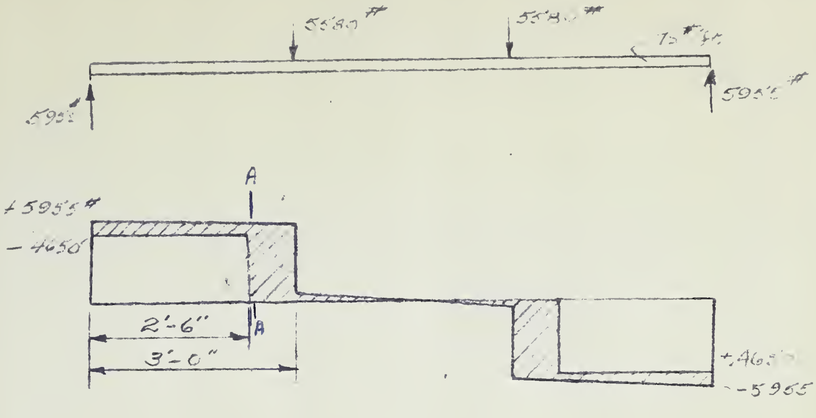
Elevation Showing Reinforcing Pattern

Fig. 3

Check on Shearing Force

While it may be said that the design is being checked against failure due to shearing forces, in reality the concern is the associated diagonal tension. Generally speaking, pre-stressing concrete provides for this without the necessity of stirrups. In any case diagonal tension is always appreciably reduced.

The significance of shear and diagonal tension is best investigated by drawing the Mohr circle for any particular case.



Net Shearing Force

Fig. 4

The cross hatched area of Figure 4 shows the net shearing force acting at any particular section. The reduction for the first 2'-6" from either support is due to the vertical component of pre-stress in the sloping reinforcing wires.

This can easily be calculated, since the arrangement of wires chosen results in a loss of eccentricity of 2.2" in 3'-0" (i.e. 3'-0" from the end of the beam).

Thus

$$R = \frac{2.2}{36} \times 66{,}200 = 4{,}050\#$$

This component acts oppositely to the shearing force caused by the loading terms, and hence results in a net shearing force of smaller magnitude. The critical section, therefore, insofar as shear is concerned is the one marked A-A in Figure 4 -- that is a section just inside the point where the reduction of eccentricity commences.

Forces Involved

Having thus determined that section A-A is critical for shear, it is now necessary to consider elements taken vertically on that section and investigate for the combination of horizontal shear and corresponding value of normal thrust (due to the pre-stress) which produces the case of maximum diagonal tension. In Fig. 5 values are plotted showing the variation of these two force components respectively over section A-A.

Rather than investigating several elements (which is a trial and error solution) it may be expedient to take the minimum value of normal thrust and combine it with the maximum value of horizontal shear. Although these values are for two different and widely separated elements, (viz. the top element for the case of normal thrust and middle element for horizontal shear) if the resulting diagonal tension is considered permissible then since any other possible combination would produce a materially smaller value, the design can be considered satisfactory without further investigation.

If, however, the resulting diagonal tension is found to be excessive, then it may be deemed wise to investigate elements selected at various distances above the center of the cross section.

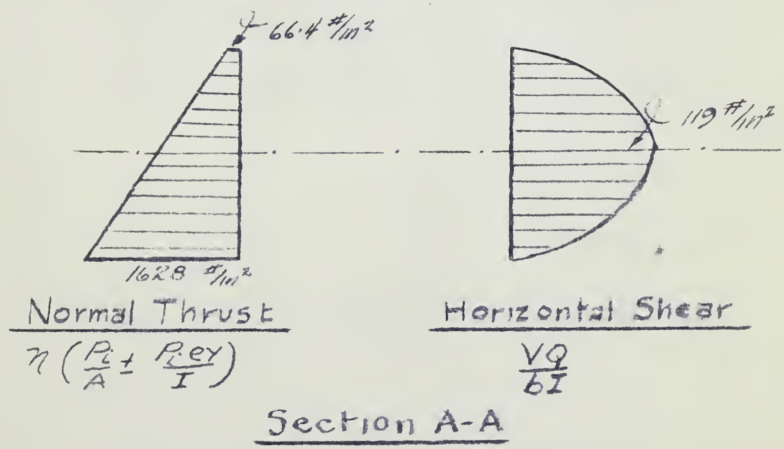


Fig. 5

The procedure outlined in the foregoing was followed in this design.

$$\text{Dead load shear} = 75\left(\frac{10}{2} - 2.5\right) = 188\#$$

$$\text{Live load shear} = \underline{5580\#}$$

$$5768\#$$

$$\text{Max. horizontal shear} = \frac{3}{2} \times \frac{5768}{72} = 120\#/\text{in}^2$$

Min. horizontal thrust due to pre-stress

$$= \frac{\eta P_i}{A} \quad \frac{e\, y_1}{r^2} - 1 = \frac{.85 \times 66{,}155}{72} \quad \frac{2.17 \times 6}{12} - 1$$

$$= 66.5\#/\text{in}^2$$

The Mohr circle is shown for this system of forces in Fig. 6.

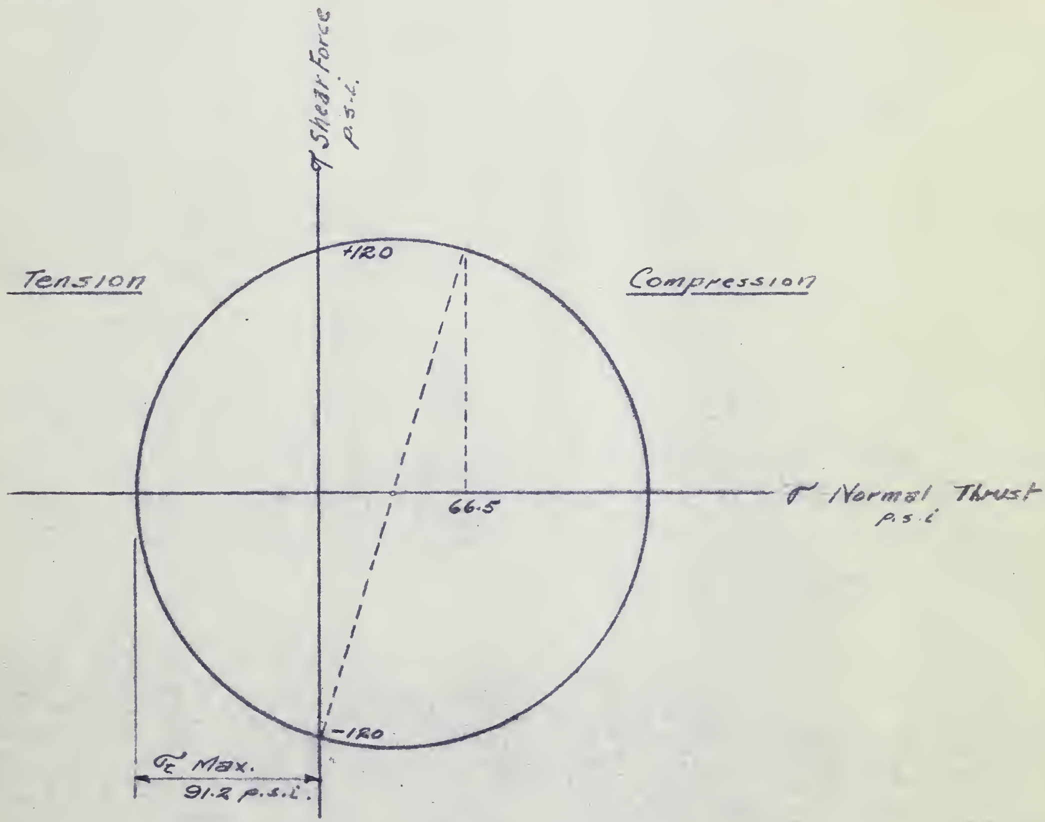


Fig. 6

The maximum tensile stress

$$t = \frac{66.5}{2}^{2} + 120^{2} \quad - \frac{66.5}{2}$$

$$= 91.2 \text{ p.s.i.}$$

By C.E.S.A.[1] specifications, the maximum allowable tension is $0.03f'_c$ or 150 p.s.i.

Thus the design is considered amply safe against failure by diagonal tension at the design load.

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(1) Canadian Engineering Standards Association

TEST PROCEDURE

A. Construction of the Test Beams

It was decided to construct two practically identical, pre-tensioned beams in accordance with the design figures arrived at in the preceding section.

To do this necessitated the design and fabrication of two sets of steel forms which could be used in this and subsequent investigations. Refer to Appendix "D" for a discussion on the factors affecting their design, and for a set of detailed drawings of the forms.

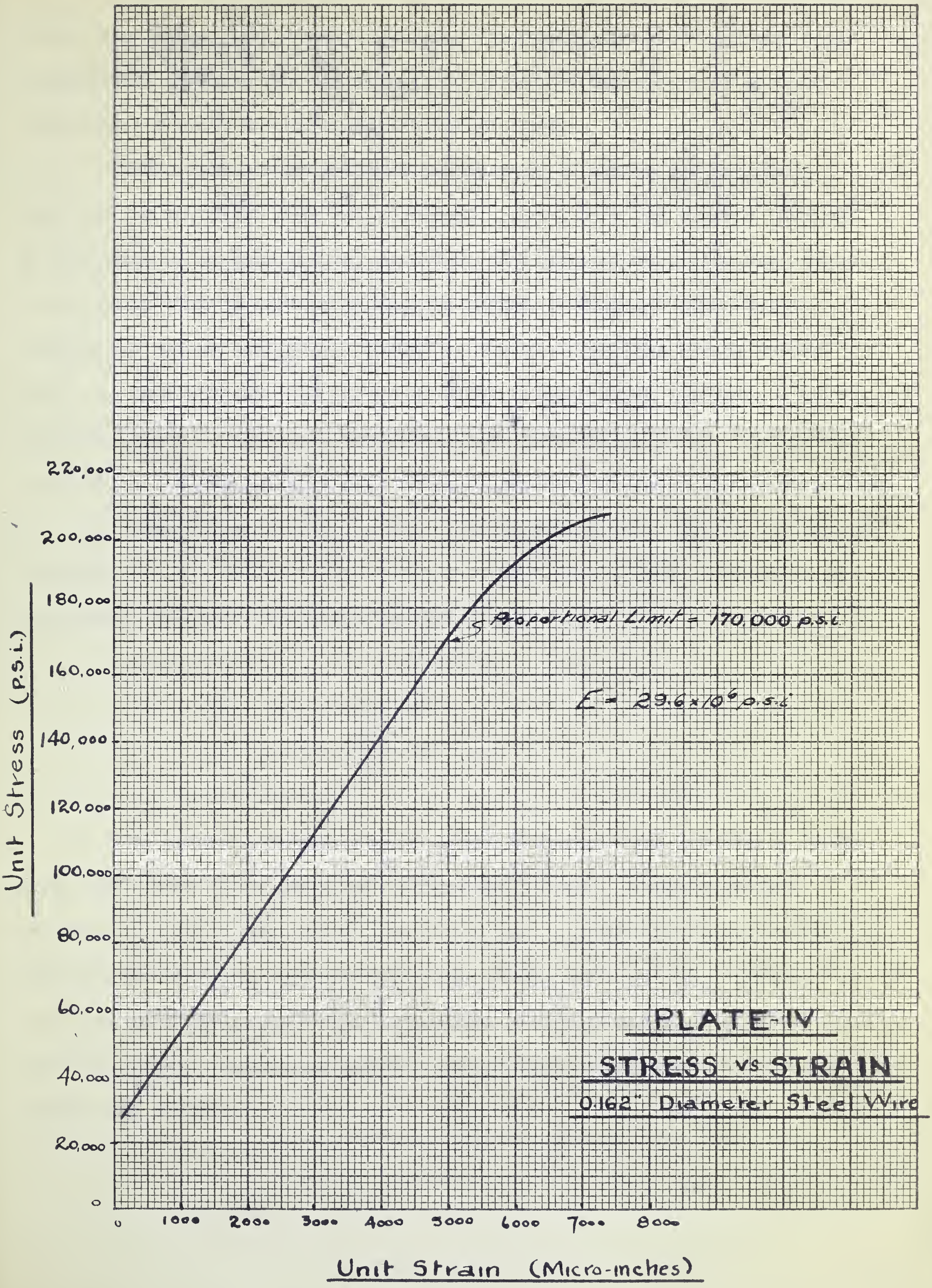
Pertinent data on the two materials used in the construction of the beams is now given.

Reinforcing steel: Was in the form of cold drawn, high tensile strength wire (supplied by the Steel Company of Canada).

The diameter of the wire was 0.162 ins. and hence had a cross-sectional area of 0.0206 sq. ins.

A loading test was performed on a sample of the wire from which E (Young's Modulus) was determined to be 29.6×10^6 p.s.i. (A discussion on this follows later - Page 65). The proportional limit was found to be 170,000 p.s.i. The stress-strain curve drawn from the results of this test is presented on PLATE IV.

Concrete: High strength concrete (5000 p.s.i.) was used. Preparatory to the actual construction of the beams, a concrete mix was designed and trial cylinders cast. Adjustment of the mix was made on the basis of the cylinder strengths. In this regard it is emphasized that the stated strength is based on standard 6" x 12" cylinders and not on cube strength. The latter gives a value of about 128% of that obtained from cylinders for the same mix.


220,000
200,000
180,000
160,000
140,000
120,000
100,000
80,000
60,000
40,000
20,000
0
Unit Stress (p.s.i.)
0
1000
2000
3000
4000
5000
6000
7000
8000
Unit Strain (Micro-inches)
Proportional Limit = 170,000 p.s.i.
E = 29.6 × 10^6 p.s.i.
PLATE IV
STRESS vs STRAIN
0.162" Diameter Steel Wire

Initially it was planned to select for study, one wire from each of the six layers. To these six wires, wire-resistance strain gauges were to have been attached for the purpose of revealing the stress at various locations and under differing magnitudes of load.

This rather over ambitious arrangement of gauges was modified when work on the first beam was commenced. Apart from the expense incurred by the usage of so many gauges, practical difficulties were also encountered. It was found that in certain areas, the number of lead out wires was excessive and became unwieldly. Secondly, the waterproofing became rather bulky, so that with six such gauge housings located close together, a considerable amount of concrete would have been displaced. It was feared that the combination of these two factors might produce an abnormally weak section.

Thus it was finally decided that only four wires should have gauges attached to them. (It is now thought that the extra gauges would not have contributed much additional information in any case).

It was desired to know the stress in these wires at

(a) their extreme ends

(b) points under each load

(c) mid-span

In order to prevent any curvature (inherent from its manf.) in the wire from rendering an inaccurate stress evaluation, it was considered essential to locate <u>two</u> gauges, end to end and on diametrically opposite sides of the wire, at any point where the stress was required. (Such a location, with <u>two</u> gauges, will hereinafter be termed a "gauge point"). From the average strain recorded at a gauge point, it was thus possible to compute the stress.

This procedure is in accordance with normal laboratory practise -- for example a Huggenberger Extensometer is used to measure strain over two gauge lengths on diametrically opposed sides of a specimen.

The method of attaching and waterproofing the gauges used in this investigation is described in detail in Appendix "C". Photographs of a typical gauge point are also shown.

A code was used to make reference to any particular gauge an easy matter. The wires selected for study and their designation, together with the location of the gauge points and their designation are shown on PLATE V. Thus the gauge points selected for wires U, X & Y were

(1) at a point below each load .... (designation P & W)

(2) at midspan .... (designation C)

(3) at either end .... (designation N & S)

For wire Z, gauge points were located as in (1) and (2) above and at both ends. Furthermore, wire Z was selected specifically to enable a study of bond to be made. For this reason, so-called "satellite" gauge points were selected, in addition to those already mentioned. At these points only one gauge was attached. Thus, although the satellite gauge points were not considered suitable to give necessarily true stress evaluations they served to indicate any sudden occurrences such as might result with a loss of bond. Six satellites were used in the case of Beam #1. Since it was to be tested without anchors, the investigation of bond was of relatively greater importance, than for Beam #2 on which the anchors were to be retained.

It was found expedient to stretch the wires over a long bench, each end being held by a vise, during the gauge attaching and waterproofing processes. Upon transferring the wires, with the completed gauge points, to the steel forms, care was exercised to avoid damage to the waterproofing. The wire had a strong tendency to return to its originally coiled shape, and hence had to be kept continually under tension during the transferring procedure.

[illegible]

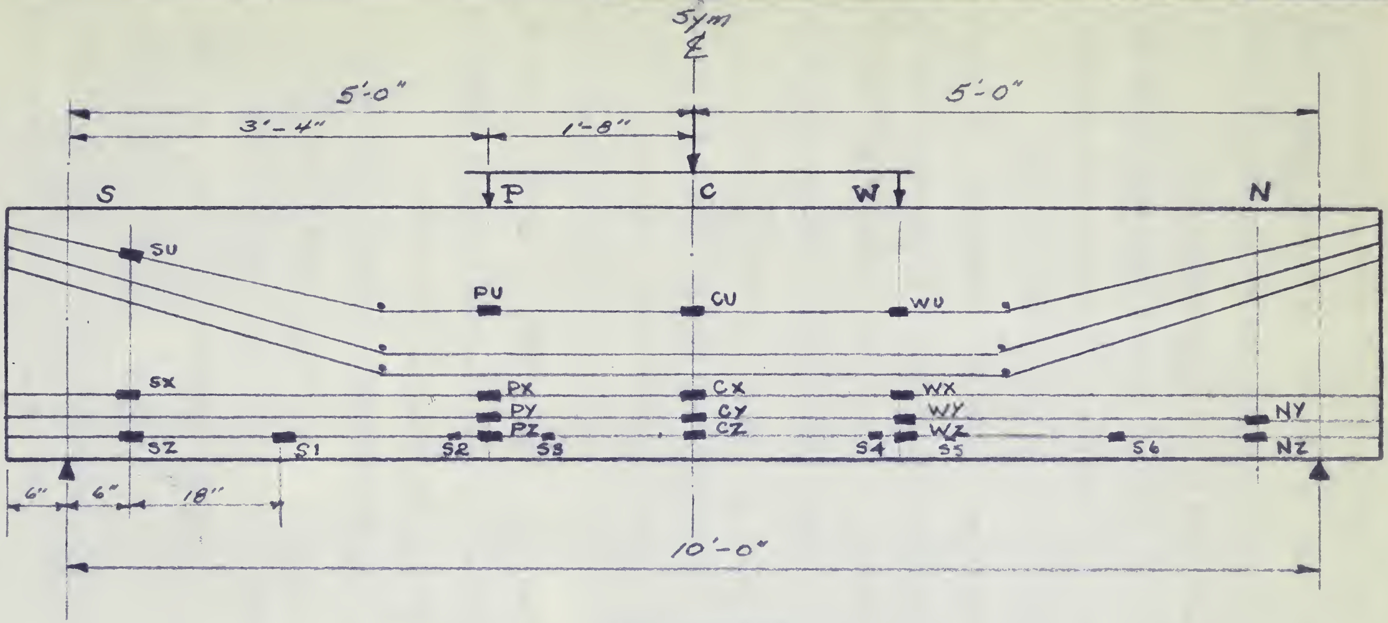
[illegible]

[illegible]

[illegible]

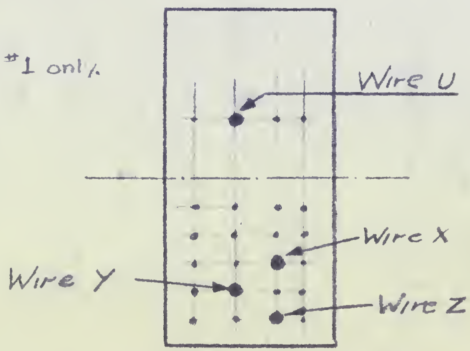
[illegible]

[illegible]



Elevation

Note:- Satellite gauges
S2-S5 incl. for Beam #1 only.



Midspan Section

PLATE V

LOADING SYSTEM

GAUGE LOCATIONS

Once the wires were in place in the appropriate holes of the steel end plates, a gripping device was placed loosely over each end. The wire was positioned longitudinally in the forms such that all gauge points were in their proper positions. At one end, sufficient length of wire was left protruding to provide for gripping in the subsequent tensioning process.

All wires were placed in the forms in this manner, starting with the bottom layer and working upwards. The dowels about which the three top layers of wire were bent, were placed during this operation.

When all wires were in place, initial or "zero" readings were recorded for the gauges. In this regard "zero" stress in the wires was taken as that condition of stress existing when the wires were pulled as straight as possible by hand. This should not introduce an appreciable error in view of the ma gnitudes of stress eventually realized.

The stressing of the wires was accomplished by means of an hydraulic jack and a section of 5" pipe arranged as shown in Figure 8.

The wires were tensioned in pairs -- each pair being made up of two correspondingly located wires either side of the horizontal centerline of the end plate. To ensure equal loads being placed on each wire it was essential to locate the jack centrally between the two wires.

A pressure gauge was attached to the fluid reservoir of the jack which provided a means of checking the approximate load on the wires. For closer determination, and for wires to which strain gauges had been attached, the matter was simply to read the strains recorded by the gauges and convert this to stress. The majority of the wires, however, had no gauges, and their stresses were determined indirectly by determining their frequency of vibration. For a full description of the apparatus and development of necessary theory used in this method see Appendix "B".

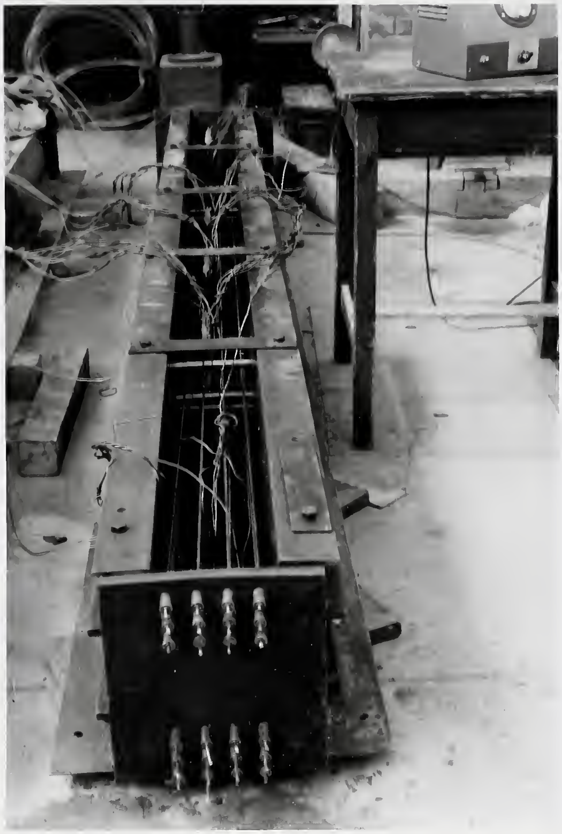


Fig. 7

General view of forms with tensioned wires in place.

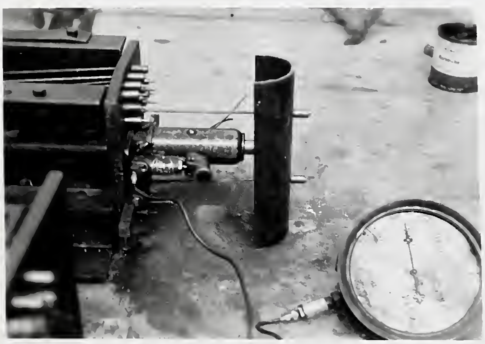


Fig. 8

Tensioning Apparatus

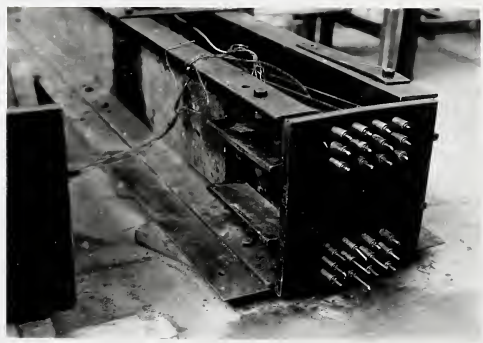


Fig. 9

End Details

As a last step before placing the concrete, all wires were cleaned with solvent to remove any grease picked up from the forms.

Figure 7 shows a general view of the wires in place in the forms after tensioning. Figure 9 is a close-up of the end plate and anchors.

The high strength concrete required meant that a rather dry stiff mix had to be handled. Thus it was essential that some method of compaction be used. Since the space between wires was limited to approximately one inch, the use of a vibrator of the type that works in the concrete was not permissible. An ordinary pneumatic jack hammer was used on the sides of the forms and the concrete was placed satisfactorily without holes or honeycombing.

B. Method of Testing

Beam #1 was selected for test without end anchorages. At this stage, considerable doubt arose over the effectiveness of bond in maintaining pre-stress. A cautious approach was therefore adopted. Anchors were removed firstly from wire "X".

Their removal was effected by placing the end of a steel bar against the anchor and delivering a sharp blow to the opposite end of the bar by means of a sledge hammer. This resulted in a cleanly sheared break of the wire with no undesirable bending placed on it which might have affected the bond.

The only loss in stress recorded on wire X was at the end N. This loss was anticipated since N was so close to the end of the beam that a sufficient length of wire was not available along which bond could develop. Accordingly then, it was decided to continue with the removal of the anchorages. Wire U was selected next. The results for it were very disappointing. An immediate loss of all useful pre-stress at all gauge points along the wire, except at W was recorded. It was decided to dispense with further

removal of the anchorages, lest the beam be rendered incapable of sustaining any appreciable load. The plan, then, was to test the beam by loading to the design load with the remaining anchors left in place, release the load, remove all anchors, and proceed with cyclic loading to failure. The chance of an immediate failure with little or no information contributed by a beam on which much time and money had been spent was thus avoided.

Since the beams had been cast one week apart there was ample time available for the testing of Beam #1 before the curing period of Beam #2 was complete.

Reaction type loading was effected by means of a Riehle Testing Machine (capacity 150,000#). The apparatus was arranged to apply two equal concentrated loads at the one-third points of the span. A general view of this appartus with Beam #2 in place is shown in Figure 10.

Centerline deflection for both beams was measured by means of an Ames dial. Rulers with graduations to the one-hundredth of an inch were used to check deflection of points two feet from each reaction.

Thus by subjecting each beam to identical loads (at least for the range of loading withstood by the weaker beam) it was possible to investigate the extent to which bond alone was reliable in maintaining pre-stress.

Presentation, Analysis and Interpretation of Test Data.

A. General

Throughout the curing period, a running check was kept on the stress in the four selected wires by recording strain gauge readings at frequent intervals. This data is presented for Beams 1 & 2 on PLATES VI & VII respectively as graphs of Steel Stress vs. Time of Cure.

During the actual testing period, record was kept of

(a) total applied load

(b) strain gauge readings

(c) deflection at the centerline and at the two outer one-fifth points

(d) temperature

at regular increments (and decrements) of applied load.

The centerline deflection characteristics are indicated for Beams 1 & 2 on PLATES VIII & IX respectively as Load-Deflection curves. The deflections of the points two feet from the supports were read to rather a poor degree of precision and the data was considered of insufficient consequence to warrant presentation here.

PLATES X to XVII inclusive, depict the trend of the strain (and hence stress) at the different gauge points under varying conditions of load. It should be noted that the horizontal scale of these Steel Stress vs. Total Applied Load graphs is not a normal constantly increasing scale, but is "stretched out" as it were, in order to follow the cyclic loading pattern. In this way it was hoped that a clearer picture of the stress tendency would be revealed. With a normal scale the picture is confused by the crossing and recrossing of the curves.

In addition to this data, a special point was made to take strain gauge readings before and after (a) the removal of the forms (b) the removal of any of the anchorage devices, in order to reveal the effect these operations had upon the stress in the wires. These results are incorporated with either the "Steel Stress vs. Time of Cure" or "Steel Stress vs. Total Applied Load" graphs, according to the sequence of events.

Interpretation of Results

Steel Stress vs. Time of Cure

PLATES VI & VII

Beam #1

With one exception (wire Y) all of the curves follow the same general pattern. Immediately after tensioning, the stress in any particular wire is seen to decrease very rapidly. This continues over the first twenty hours. The loss in stress can very probably be attributed, for the most part, to plastic flow or "creep" of the steel wire itself.

Following this period of loss of stress an "apparent" recovery of stress is indicated. For this early portion of the curing period the strain gauges are perhaps none too reliable, due to the shrinkage and thermal effects of the newly placed concrete. It is thought that herein lies an explanation for the apparent stress recovery. The concrete in shrinking would cause pressure, (more or less uniformly distributed by the asphalt housing) to bear upon the gauges. Such a pressure would produce the same effect on the gauges as that observed for an actual increase in stress. In any case this irregular behavior is undoubtedly due in some way to the action of the concrete. This is substantiated by results of a test performed on a wire stressed in the forms with no concrete placed about it. The trend is towards a continual loss in stress at a decreasing rate of loss, with no stress recovery indicated.

After the first three days, with the exception of wire Y, the gauges indicate that no radical stress changes occurred, and remained for the most part practically constant. In view of the steadiness in gauge readings for all other wires including those of Beam #2, it is considered

that the erratic nature of those on wire Y was due to one or more faulty gauge. In fact, readings of only two of the four gauge points were considered of sufficient accuracy to warrant use.

Initial values for wire U are missing due to a broken common lead that was not immediately located.

Beam #2

The curves for Beam #2 agree in general with the pattern set by those for Beam #1 and are exceptionally consistent with one another. No explanation is offered for the improvement of results for Beam #2, unless the experience gained in fabricating the first is reflected in the results of the second.

PLATE VII shows that a sharp drop in stress occurred during the first few hours, followed by a slight recovery (much smaller than the case of Beam #1). For the remaining period all wires showed an almost constant stress, or perhaps at most a very slight uniform decrease.

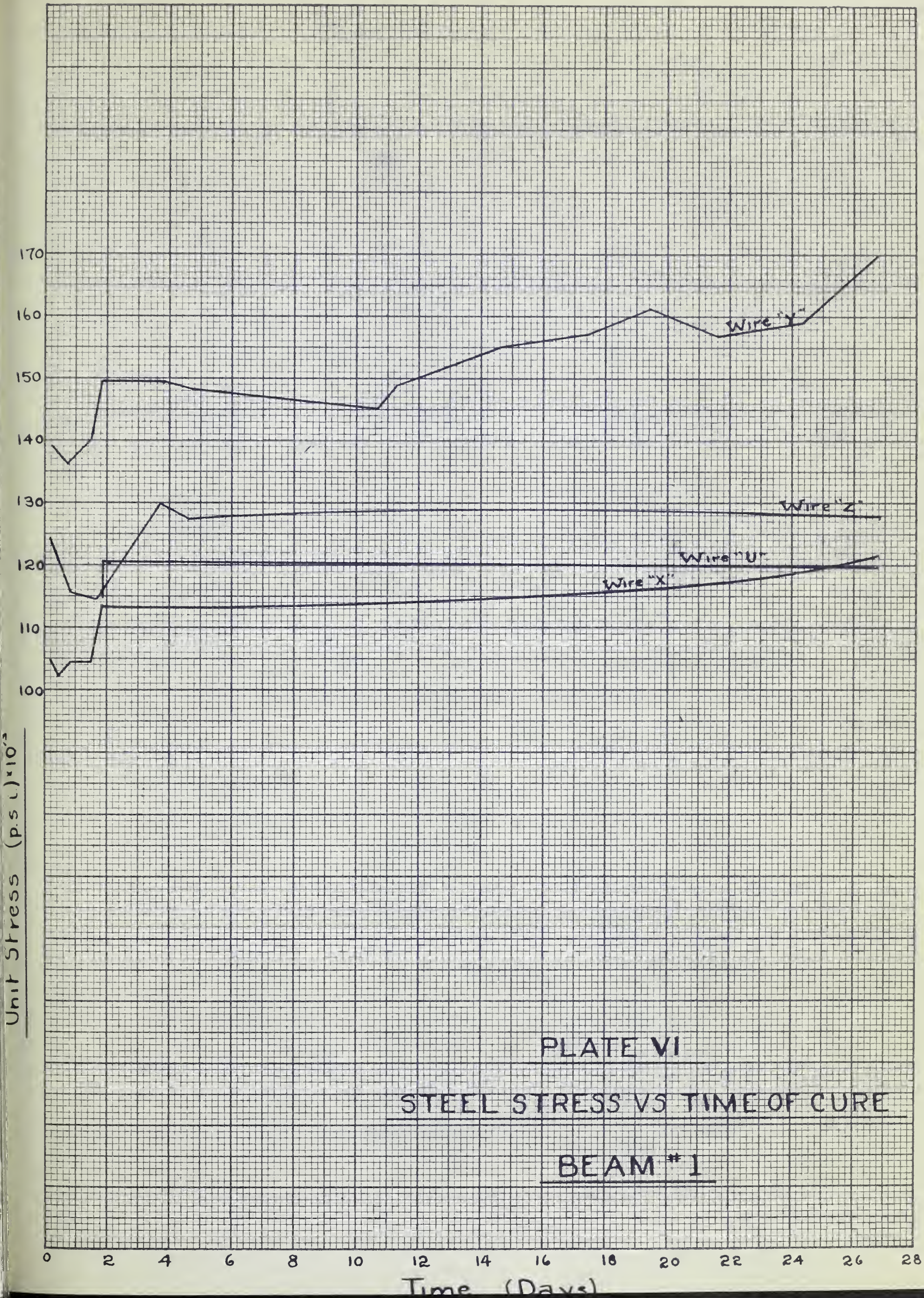


PLATE VI

STEEL STRESS VS TIME OF CURE

BEAM #1

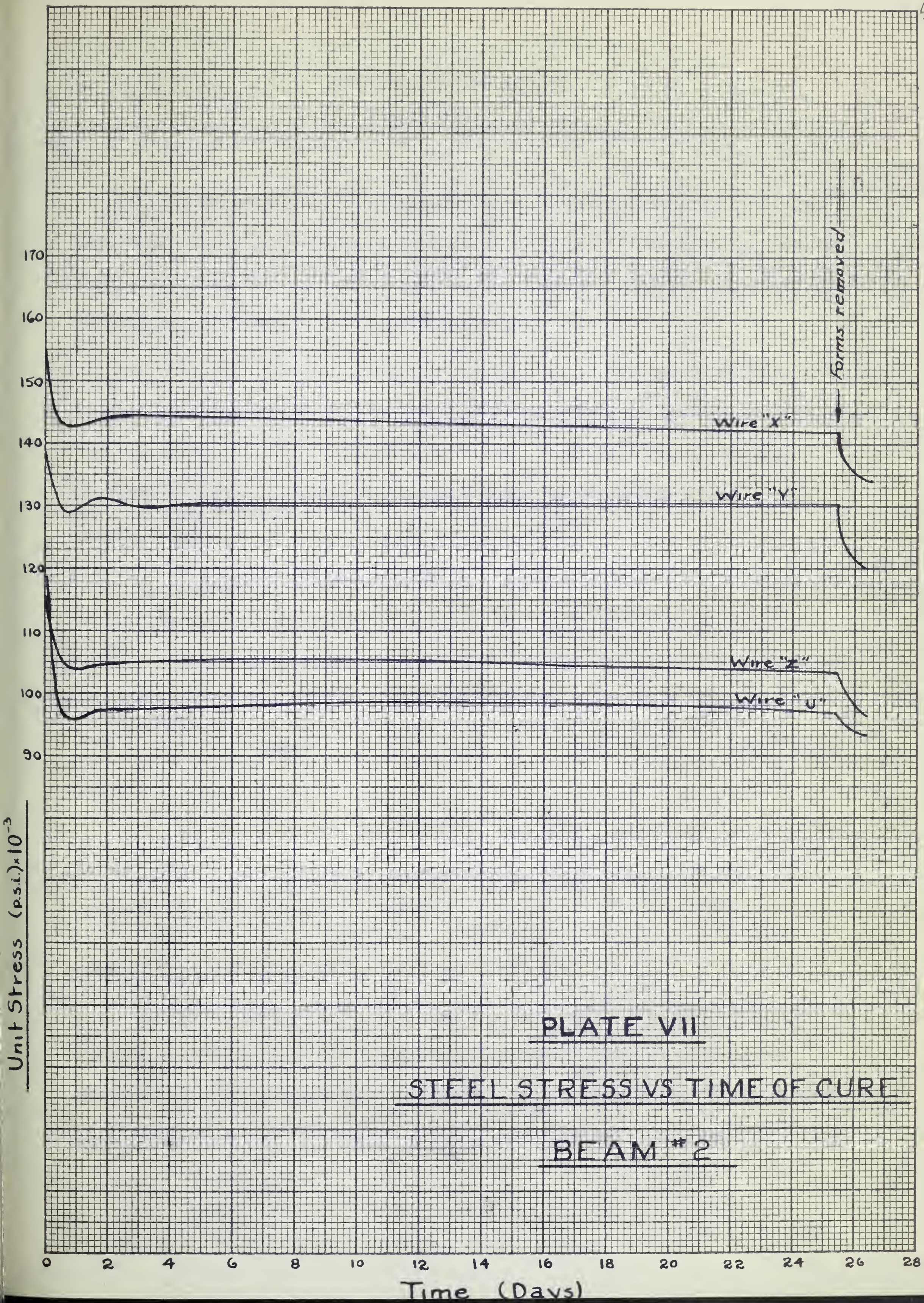


PLATE VII

STEEL STRESS VS TIME OF CURE

BEAM #2

Load-Deflection Curves

PLATES VIII & IX

Both beams gave "hysteresis" type curves under cyclic loading. This would serve to indicate a failure on the part of the beams to recover completely from the deflected state as the load was reduced. However, two factors greatly affect the degree of recovery.

Perhaps the most important of the two is the length of time allowed between observations of deflection. In other words, at least a portion of the remaining deformation is simply due to a lagging effect with respect to time. This is illustrated by the recovery made overnight while under constant load.

The second effect results in a permanent deflection experienced due to the lack of sufficient energy to provide a restoring force. This arises out of a dissipation of some of the energy used in initially causing the deflection. The effect is analogous to the hysteresis loss observed when a magnetic material is subjected to cyclic reversals of a magnetizing force.

In the case of a pre-stressed beam where the percentage of steel is small, this characteristic is considered indicative of the fact that concrete is not a perfectly elastic material.

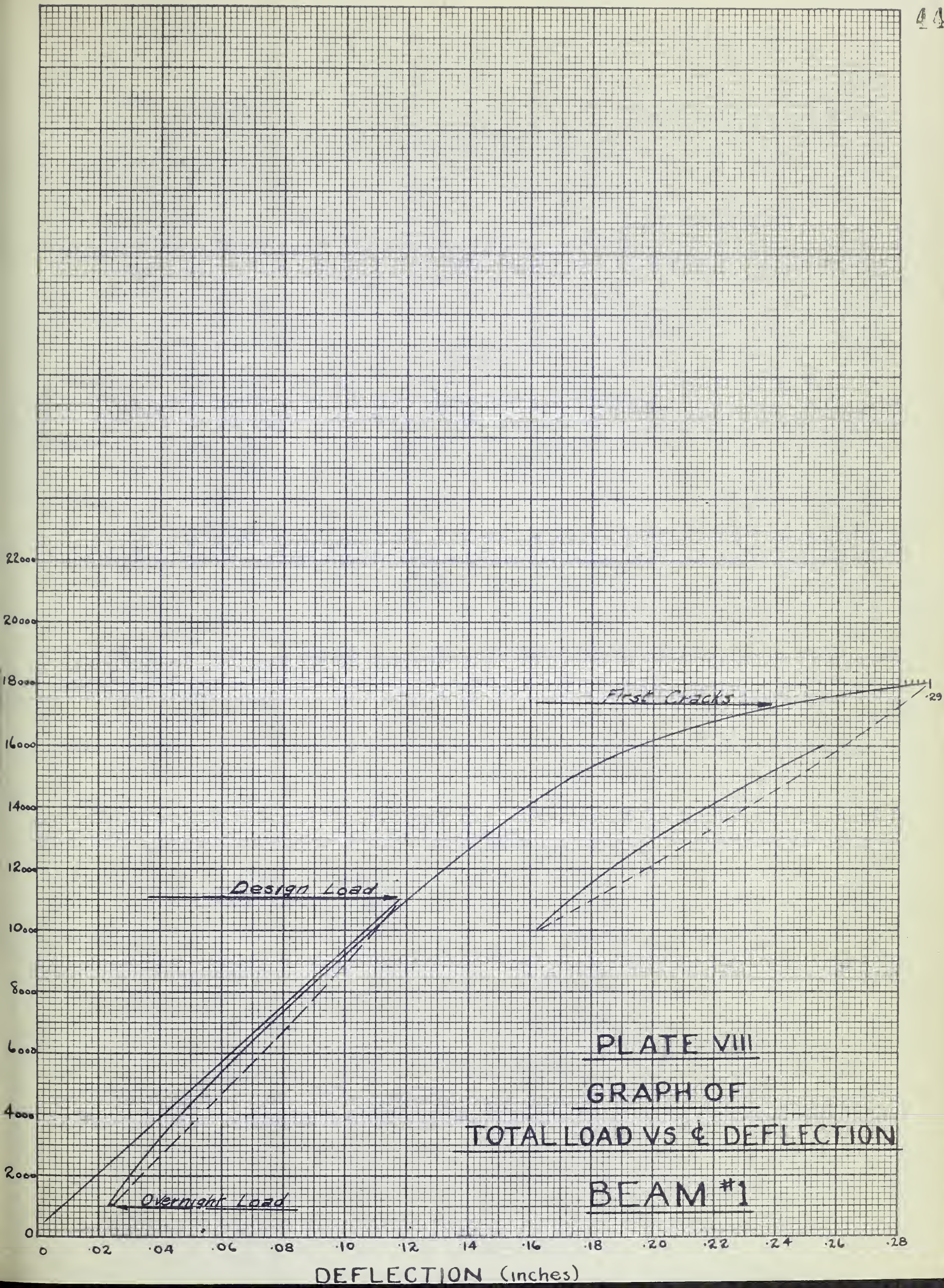
Beam #1

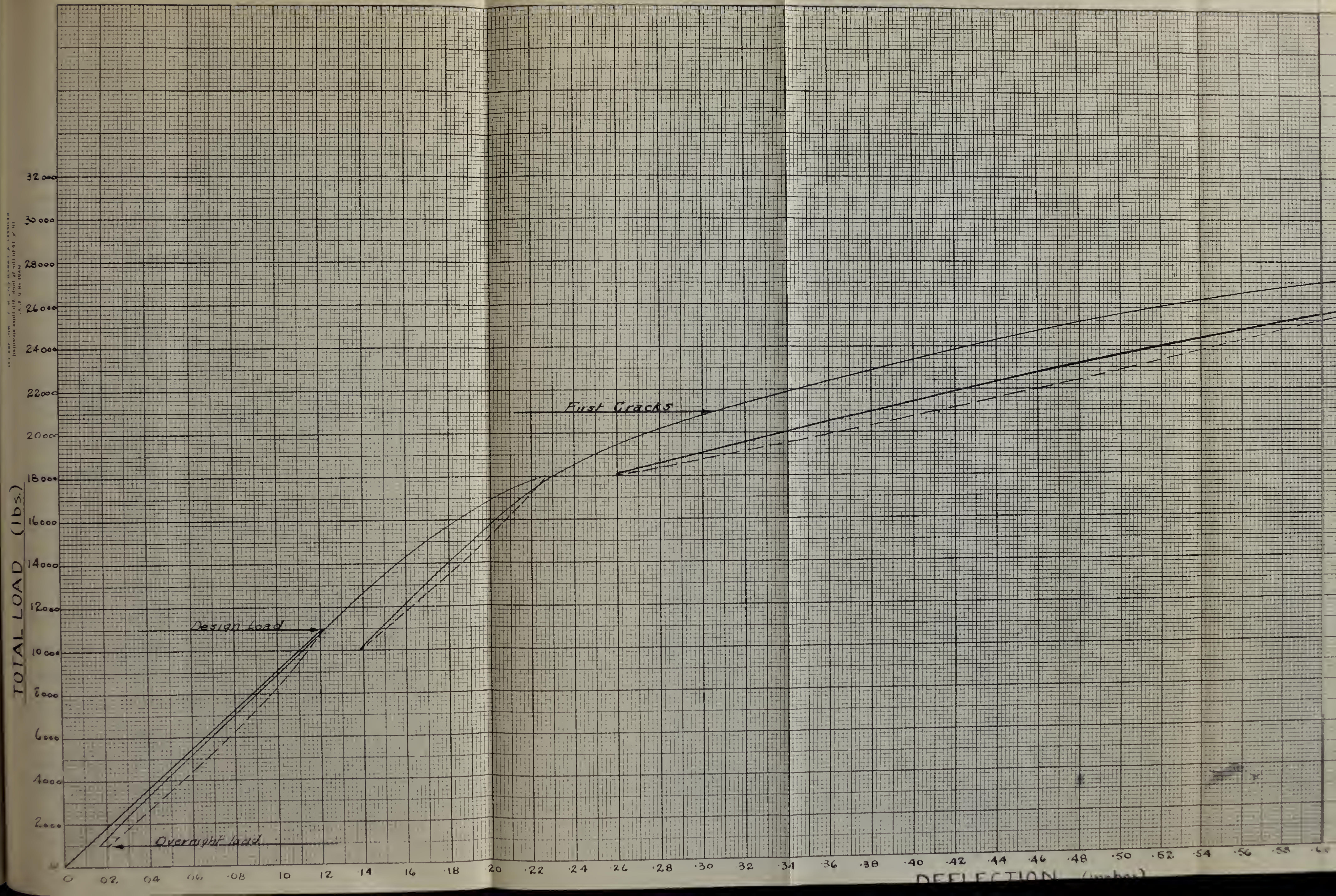
From PLATE VIII it will be seen that under the initial loading to 11,000#, and with all but two of the wires still anchored mechanically by means of the gripping devices, a straight line relationship exists between load and deflection. Furthermore it appears that this characteristic would

have extended at least as far as the design load, had the initial loading been performed on the beam stripped of the anchors. This is suggested by the graphical results of the second loading, at which time all the anchors had actually been removed.

Beam #2

PLATE IX shows the straight line relationship to exist between load and deflection to a total load of slightly greater than 13000#. This represents an increase of approximately 16% over Beam #1. It is also apparent on comparison of PLATES VIII & IX that Beam #2 exhibited superior recovery properties -- that is, it behaved more elastically.


22000
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0
.02
.04
.06
.08
.10
.12
.14
.16
.18
.20
.22
.24
.26
.28
.29
First Cracks
Design Load
Overnight Load
DEFLECTION (inches)
PLATE VIII
GRAPH OF
TOTAL LOAD VS ℄ DEFLECTION
BEAM #1



TOTAL LOAD (lbs.)
32000
30000
28000
26000
24000
22000
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
.02
.04
.06
.08
.10
.12
.14
.16
.18
.20
.22
.24
.26
.28
.30
.32
.34
.36
.38
.40
.42
.44
.46
.48
.50
.52
.54
.56
.58
.60
First Cracks
Design Load
Overnight load
DEFLECTION (inches)

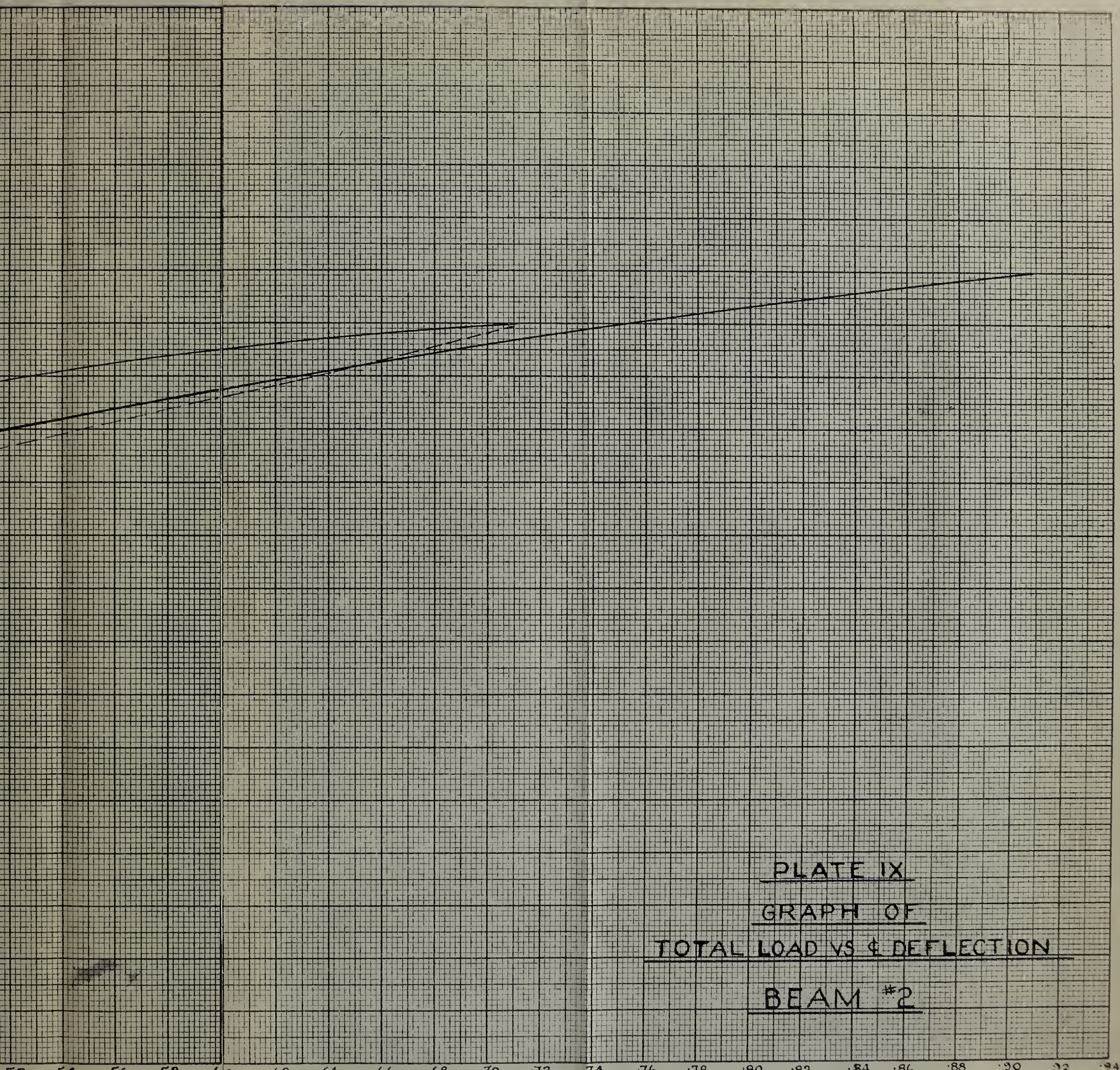


PLATE IX
GRAPH OF
TOTAL LOAD VS ¢ DEFLECTION
BEAM #2
·52 ·54 ·56 ·58 ·60 ·62 ·64 ·66 ·68 ·70 ·72 ·74 ·76 ·78 ·80 ·82 ·84 ·86 ·88 ·90 ·92 ·94

Steel Stress Vs. Total Load

PLATES X to XVII incl.

These curves show graphically, the stress variation in the reinforcing wires during the loading cycles. In interpreting the results of these graphs, the location of the particular wire and gauge point under consideration must be kept clearly in mind. PLATE V indicates these locations with respect to loading points, reaction points and neutral axis.

Beam #1

(a) Wire U (Refer to PLATE X)

Prior to loading the beam, in fact prior to the removal of the steel forms, the end anchorage devices of wire "U" were sheared off. Immediately following this, the strain readings were recorded from the gauges on this particular wire. The results indicate a complete loss of useful stress throughout most of the length of the wire. Only in the region of gauge point W was the tension retained. Gauge points S, P & C reveal the occurrence of great losses of stress varying from a complete drop at S to a 50% drop at C.

Further losses are indicated as having resulted upon removal of the forms. This is due largely to the elastic deformation undergone by the concrete. Although the loss is noted as being sudden in nature, some further loss is indicated as having occurred up to 24 hours later. The latter effect is probably attributable to plastic flow by the concrete.

During the loading cycles that followed, little significance is attached to the results recorded at gauge points S, P & C. It is thought that perhaps the slippage resistance of the wire, together with the wedging

action of the gauge housings was all that prevented the stress in the wire from dropping completely to zero.

With regard to gauge point W, noteworthy is the fact that for an increasing load on the beam, the stress at W drops, whereas for a decreasing load the reverse is true. This is not to be wondered at when it is remembered that gauge point W lies well above the neutral axis of the beam.

(b) Wire X (Refer to PLATE XI)

As for the case of wire "U", the anchors on "X" were removed before loading was commenced. At gauge point S an immediate loss of almost 50% of the stress in the wire was recorded. Gauges at W, P, & C, however, indicated no changes whatsoever and hence sufficient bond must have developed in order to have maintained the tension over most of the length of the wire.

As for the case of wire "U" a sharp drop in stress was experienced at all points, upon removal of the forms.

For the case of loading up to the design load, a very slight increase in stress occurred at P, W & C. This increase amounts to about 5% of the initial stress. Over the design load, however, appreciably greater increases occurred until at a load of 17,400 # (whereupon cracks suddenly appeared under loading point P) the stress at gauge point P dropped instantly. The loss sustained was over 50%. This indicates a slipping of the wire at the southern end of the beam.

In general, for increasing loads, stresses at the gauge points increased and vice versa (gauge point S excepted). This, of course, is to be expected for any point located below the neutral axis.

(c) Wire Y (Refer to PLATE XII)

End anchors were left in place on this wire until after the completion of one loading cycle up to the design load. The load was then completely removed to permit the shearing off of the anchorage devices.

As this was done, all useful stress at N was completely lost. No effect was revealed at points P, C & W. Very little more can be said of the stress tendencies, which were similar in pattern to those of wire X, except that the variations were more pronounced due to the fact that the gauge points were displaced further from the neutral axis.

(d) Wire Z (Refer to PLATE XIII)

The gauge points on this wire recorded the same type of stress variations as reported for the two previous wires. Wire Z being located farther below the neutral axis than the wires in any other layer, accordingly showed slightly more pronounced stress variations.

Upon removal of the anchorages, loss of stress was recorded at S and N only. No effect was noted on satellite gauges S1 and S6. These gauges were approximately 18 inches from S & N respectively. Thus in 18 inches or less sufficient bond was developed to maintain the full stress in the wires. The remaining satellites contributed no additional information and it was considered that no purpose would be fulfilled in plotting the results.

Beam #2

Inspection of PLATES XIV to XVII will reveal for the most part, a substantiation of the stress characteristics displayed by the wires of Beam #1. The explanations offered in the foregoing for the various occurrences apply equally well to Beam #2. A rigorous discussion on the behavior of each gauge point is therefore not deemed necessary. However, the following special points should be noted.

Because Beam #2 was capable of withstanding substantially greater loads, the wires, in all cases, were subjected to much higher stresses than their counterparts of Beam #1 This is particularly true for the wires designated X Y & Z. It should be pointed out therefore that for the upper reaches of these curves, the straight line relationship between stress

and strain no longer exists. (See PLATE IV). It is therefore necessary, when dealing with magnitudes of stress exceeding 170,000 p.s.i. to use the graph of PLATE IV to convert measured strain to stress.

Rather eratic results were observed for gauge points WX & PZ. This can be directly attributed to leaky gauges, as evidenced by the difficulty experienced in obtaining a true balance on the strain indicator because of a wavering action by the galvanometer needle. However, despite this, the resulting curves follow the general trend, expected from the results at other gauge points.

Contrary to the case of Beam #1, no anchorage devices were removed and hence no gauge points were rendered ineffective due to this cause. Beam #2 therefore presents the only record of stress variations in the wires at the ends of the beams. It will be seen from the curves for gauge points S & N that the stress there remained practically constant in all cases during the loading cycles except for a rapid rise as failure became imminent.

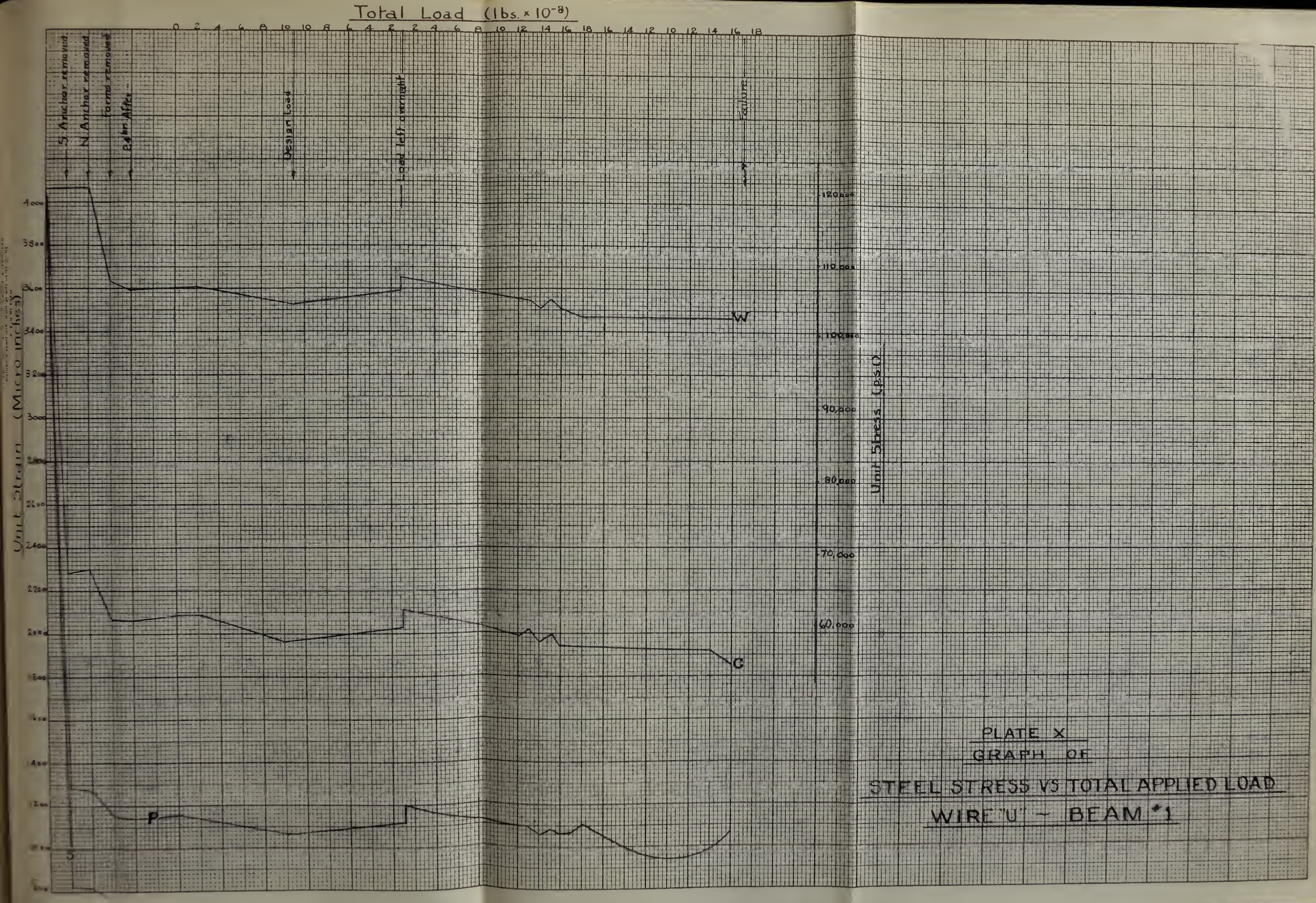
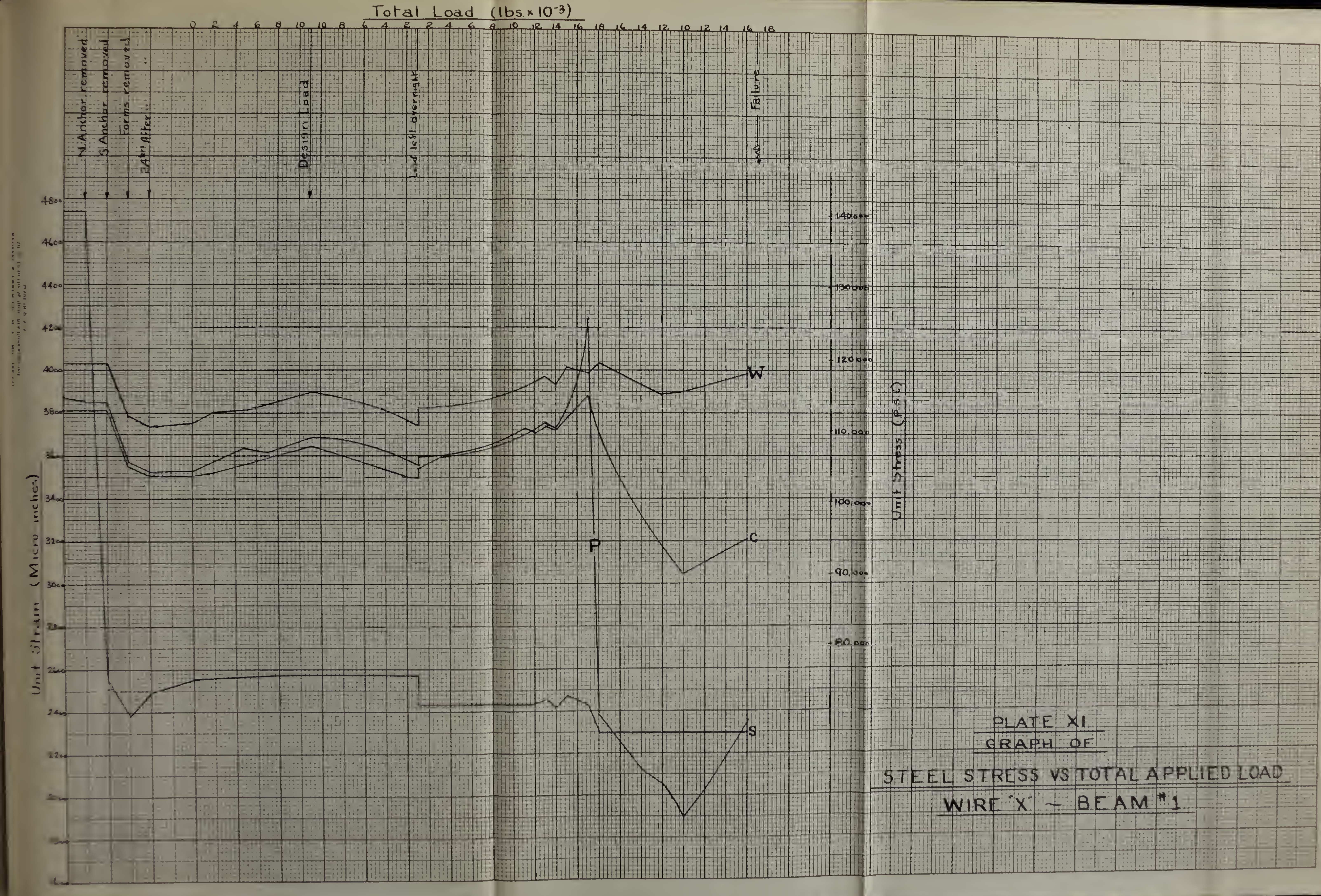


PLATE X
GRAPH OF
STEEL STRESS VS TOTAL APPLIED LOAD
WIRE "U" – BEAM #1


Total Load (lbs. × 10⁻³)
N. Anchor removed
S. Anchor removed
Forms removed
24 hrs After
Design Load
Load left overnight
Failure
Unit Strain (Micro inches)
Unit Stress (P.S.I.)
140,000
130,000
120,000
110,000
100,000
90,000
80,000
W
C
P
S
PLATE XI
GRAPH OF
STEEL STRESS VS TOTAL APPLIED LOAD
WIRE "X" – BEAM #1

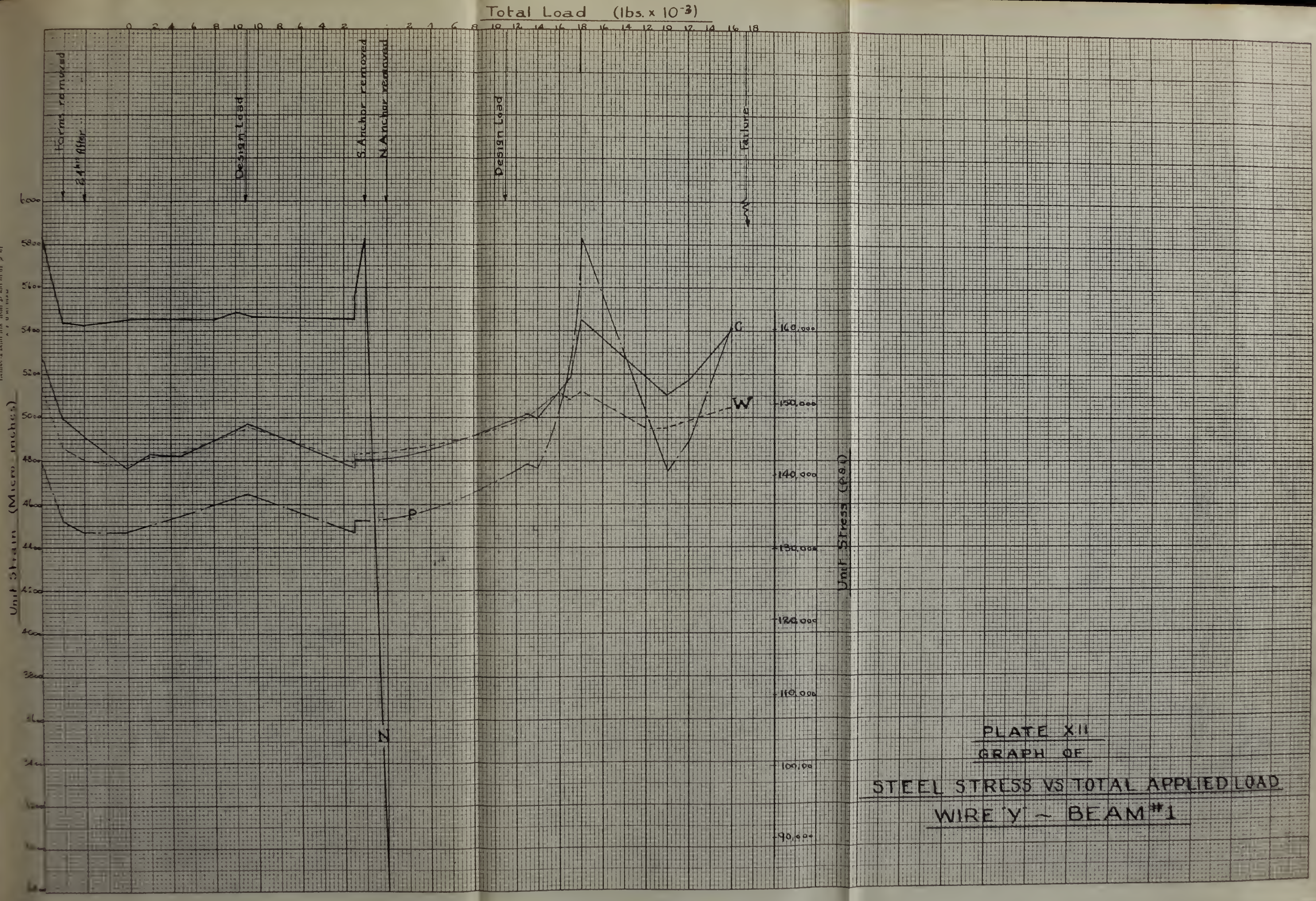


PLATE XII
GRAPH OF
STEEL STRESS VS TOTAL APPLIED LOAD
WIRE 'Y' ~ BEAM #1

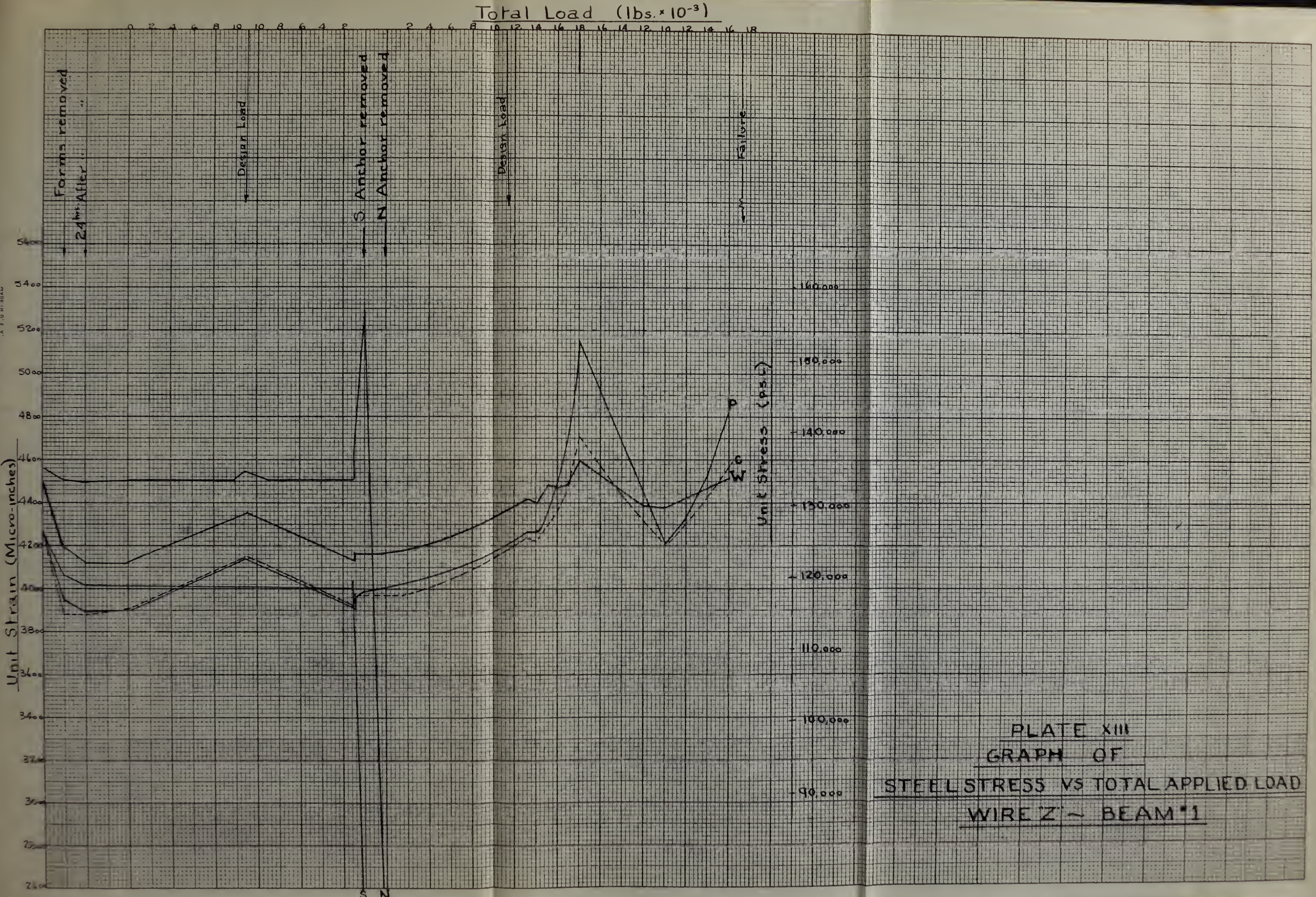


PLATE XIII
GRAPH OF
STEEL STRESS VS TOTAL APPLIED LOAD
WIRE "Z" ~ BEAM #1

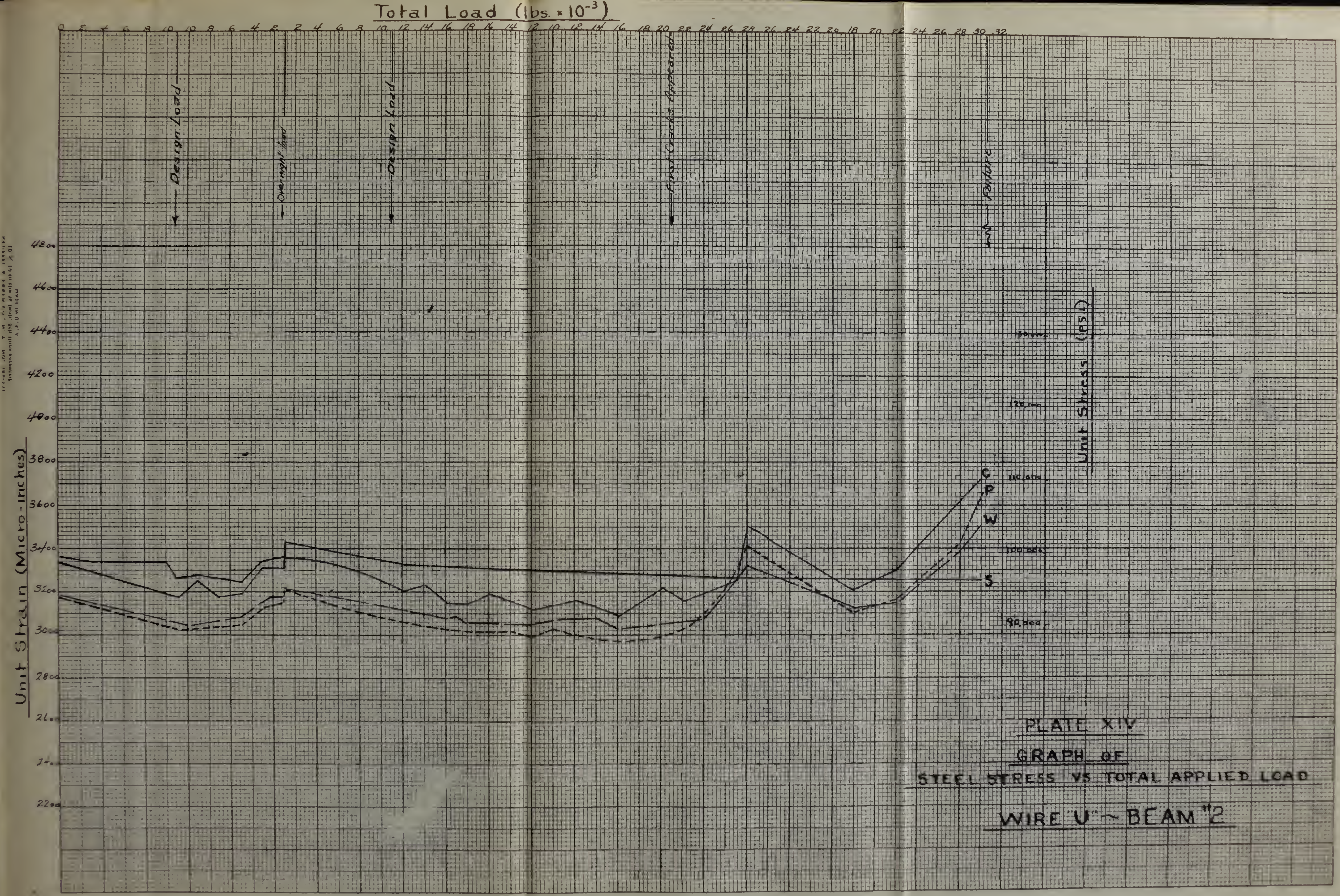


PLATE XIV

GRAPH OF

STEEL STRESS VS TOTAL APPLIED LOAD

WIRE "U" ~ BEAM #2

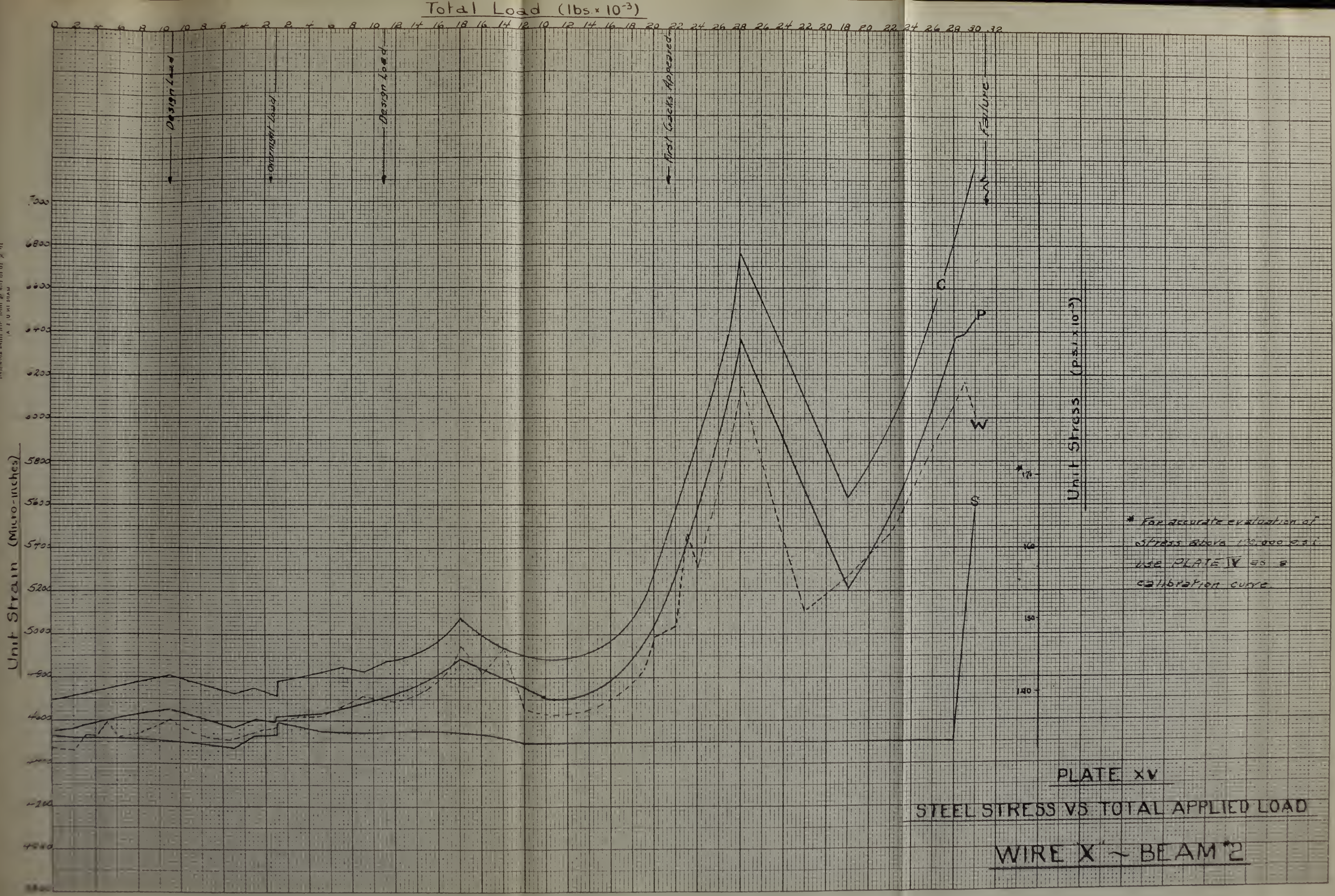
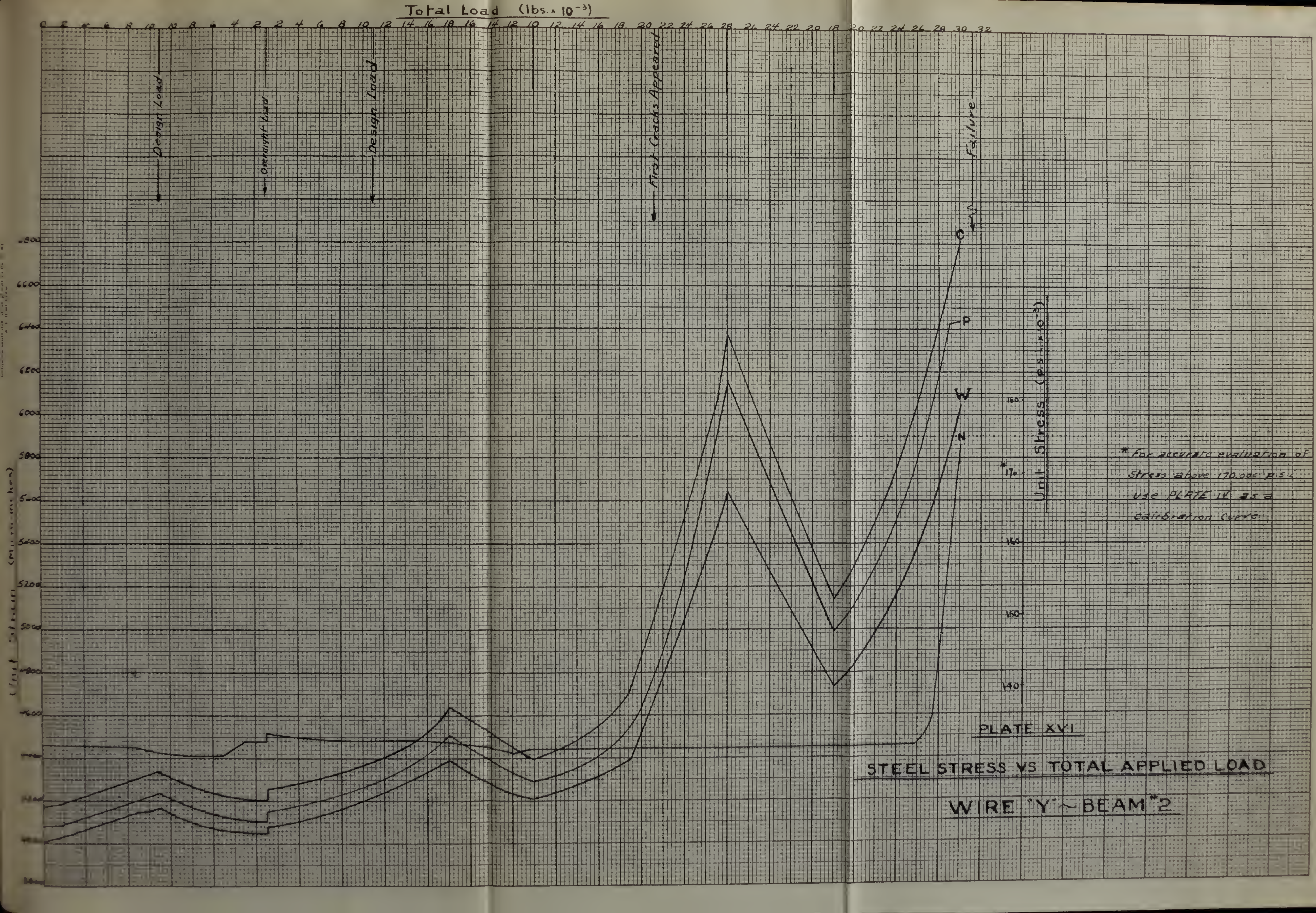
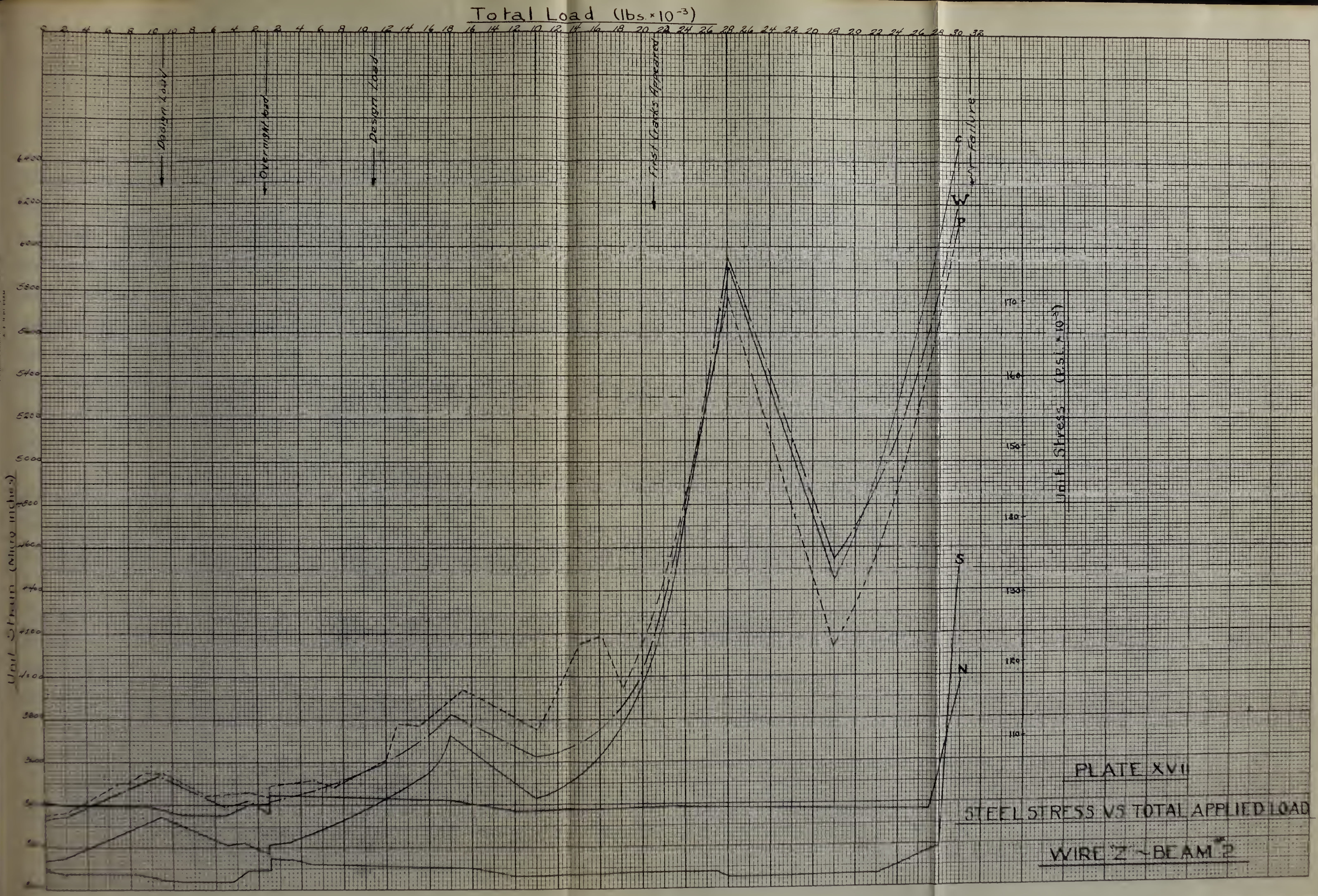


PLATE XV

STEEL STRESS VS TOTAL APPLIED LOAD

WIRE "X" ~ BEAM #2


Total Load (lbs. x 10⁻³)
Design Load
Overnight Load
Design Load
First Cracks Appeared
Failure
Unit Stress (p.s.i. x 10⁻³)
Unit Strain
* For accurate evaluation of stress above 170,000 p.s.i. use PLATE IV as a calibration curve
C
P
W
N
PLATE XVI
STEEL STRESS VS TOTAL APPLIED LOAD
WIRE "Y" ~ BEAM #2


Total Load (lbs. × 10^-3)
Design Load
Overnight Load
Design Load
First Cracks Appeared
Failure
C
W
P
S
N
Unit Strain (Micro inches)
Unit Stress (P.S.I. × 10^-3)
PLATE XVII
STEEL STRESS VS TOTAL APPLIED LOAD
WIRE 2 ~ BEAM 2

Discussion

The consideration of the manner in which each of the beams failed is of primary importance.

For the case of Beam #1, first evidence of impending failure was seen with the appearance of a crack on the bottom surface of the beam directly below the southern load point (P). This occurred at a total load of 17,400#. Loading was continued until 18,000# was sustained. Under this load appreciable widening of the crack occurred. (See Fig. 11). The load was then reduced to 11,000#, whereupon the crack closed. On subsequent reloading the crack reappeared at a lower load (16,000#) than previously. Then at approximately 17,500# complete failure took place. Further "loading" only caused increased deflection while the sustained load dropped. Figure 12 shows a general view of the condition of the beam at this point. Subsequently the load was completely removed.

Failure, unquestionably, was caused by a <u>loss of bond</u> occurring between the load point P and the nearest end of the beam. During excessive loading after failure, the pulling in of the wires at that end of the beam was plainly visible.

It is to be noted that failure occurred at a load less than the 18,000# once sustained by the beam. This serves to indicate that some of the wires must have slipped on the initial loading hence reducing the ultimate load attainable on reloading. Indeed, PLATES X, XI, XII & XIII indicate that slippage of at least some of the wires must have occurred as early as 13,000 - 14,000# on the initial loading. This is manifested in the fluctuating stress pattern for these loads.

During the loading after failure, a maximum centerline deflection of 1.67 inches was recorded for a sustained load of 11,000#. Greatest

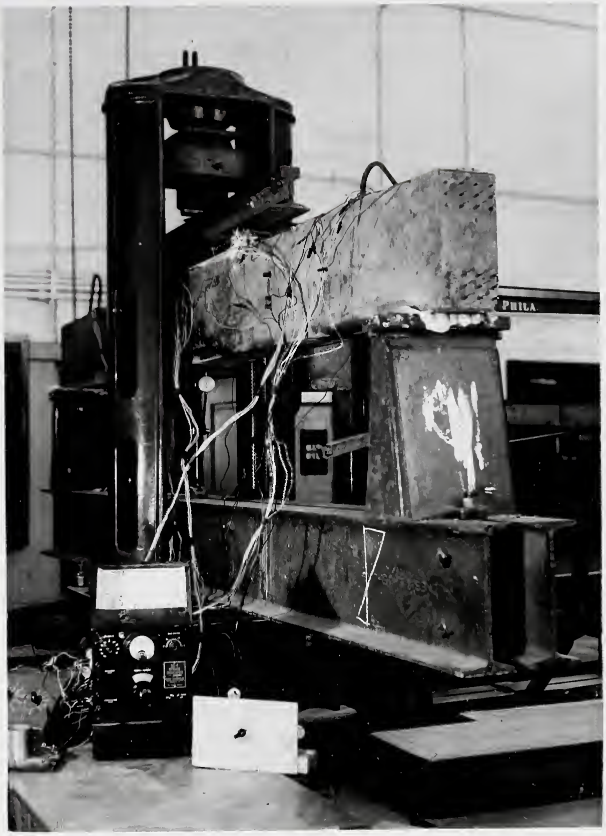


Fig. 10

Beam #1 in Riehle Testing Machine

Fig. 11

Crack at 18,000# prior to Failure

Fig. 12

Failed section (after excessive loading past failure)

deflection prior to failure was 0.29 inches. Upon reducing the load to zero, a permanent deflection of 1.42 inches was noted. This represents a recovery based on the maximum deflection of only 15%.

The safety factor for the beam with respect to the design load of 11,000# was 1.59.

The twenty-eight day cylinder strength averaged 4750 p.s.i. The cube strength obtained from three two-inch cubes cut from an undamaged portion of the beam was 6010 p.s.i. or approximately 4700 p.s.i. when converted to standard cylinder strengths.

The failure of Beam #2 was due to an entirely different cause than that for the case of Beam #1.

First cracks did not appear until a total load of 21,000# was applied. At this point a series of hair cracks became evident on the bottom surface of the beam, spread at intervals of about ten inches between the two loading points. With the application of further load, one crack immediately below load point P became greatly widened. This in effect, permitted a relaxation along the bottom surface, with the result that the remainder of the cracks tended to close up. Loading was continued to 28,000#, then dropped to 18,000#. The cracks at this latter load were completely closed. On subsequent reloading the cracks reappeared at 20,000#. Finally at approximately 31,000# complete failure took place. Some excess load was applied after failure, as for the case of Beam #1, and then all load was completely removed. After remaining in the unloaded state overnight, the design load was once again placed on the beam.

Failure in this case was due to a <u>compression failure</u> on the part of the <u>concrete</u>. This occurred near the top of the beam and under load point P. (See Figure 16).

As for Beam #1, excessive loading past failure caused increased deflection with a drop of sustained load. The maximum centerline deflection recorded was 1.58 inches, at which time the load had dropped to 14,000#. The greatest deflection before failure was 0.91 inches.

Recovery properties were excellent. On complete removal of the load after failure, the deflection was only 0.19 inches, representing a notable recovery of 88% (again based on the maximum deflection). See Figures 15, 16, 17 & 18 for pictures of the state of failure under the noted loads.

The factor of safety for the beam was 2.82.

The twenty-eight day cylinder strength was 5300 p.s.i. while cube strength was 8880 p.s.i. or converted to equivalent cylinder strength, approximately 6940 p.s.i.

It appears, therefore, that for the form of reinforcing steel used in this investigation, sufficient pre-stress can be maintained by bond alone to permit the sustainance of loads somewhat greater than the design figure. However, the factor of safety leaves much to be desired. Consequently it is recommended that in practise the mechanical anchorage devices be left in place. This is based upon the added factor of safety obtainable (2.82 as compared to 1.59) which is more nearly in agreement with the generally accepted value of 2.5 used in the design of ordinary reinforced concrete. It must also be borne in mind that the investigation has not taken into account the effect of time (say a period of years) on the loss of pre-stress. This would probably reduce further the observed safety factors.

The highly desireable recovery characteristics from chance overloading also make the retension of the anchorage devices advisable.

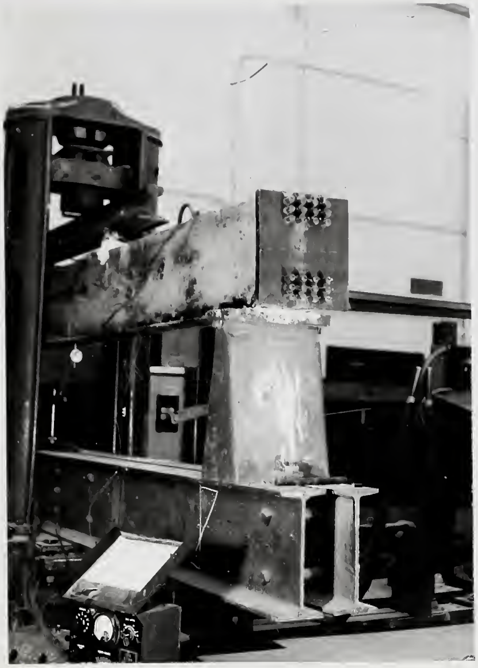


Fig. 13

Beam #2 in Riehle Testing Machine

Fig. 14

Anchorage - Beam #2

Fig. 15

Failure of Beam #2 - 31,000#

Fig. 16

Failure 31,000#

Fig. 17

Same section - zero load
(Note closure of cracks).

Fig. 18

Same section - 11,000#
(Design Load).

Sequence of loads noted is same as order of figures.

As predicted during the design, no tendency to fail by diagonal tension was observed for either beam. This is particularly noteworthy in view of the fact, that tests conducted at this institution over a period of years on ordinary reinforced concrete beams of approximately the same size and span, indicate that diagonal tension is a frequent cause of failure. It is resistance to this type of failure that constitutes one of the main advantages of this method of construction, notwithstanding the fact that many writers on the subject make no mention of it.

It has been shown, not conclusively, that for a wire of the diameter used, (0.162 inches) sufficient bond was developed along a length of eighteen inches or less, to maintain a stress of 130,000 p.s.i. At most this length, known as the transmission length, is 108 times the diameter. This is in agreement with the finding of M. R. Ros who reported that a length of 100 to 120 times the diameter was required for 0.12 inches diameter wire under a stress of 152,000 p.s.i.

For the most part, the gauge readings substantiated the beam theory. For gauge points located below the neutral axis and for loading below the design load, the change in stress in the wires was slight. Above the design load very appreciable changes were recorded depending upon the extent of overloading. (i.e. past the design load). Moreover, for any particular load, the stress variation was more pronounced for cases further removed from the neutral axis. In other words, above the design load the reinforcing steel is seen to act more or less in the manner of that in a conventional reinforced concrete beam.

For locations above the neutral axis, the stress decreased with an increasing load and vice versa. The exception to this is the increase recorded at such points for extremely high overloads and after the appearance of cracks. This does not contradict the theory. It simply means that the

neutral axis moved upwards as the effective cross-section of the beam was reduced by the upward progression of tension cracks from the bottom fibre. Thus eventually these locations found themselves below the neutral axis and behaved accordingly.

Little effect was recorded by gauges near the ends of the beam until failure was close at hand. This follows since the bending moment and hence fibre stress is negligible at such sections.

Some controversy arose over the value obtained for the modulus of elasticity for the steel wire. This value was obtained using wire-resistance strain gauges. Since gauges of exactly the same type were used in the beams, correct stresses can be obtained no matter how the value of E varies from that found elsewhere. This follows since this factor is used simply as a calibration constant for conversion of gauge readings to stress.

Conclusions

Out of the foregoing rather rigorous analysis, it seems necessary to set forth the conslusions as a number of concise statements without further discussion.

1. For the form of reinforcing steel used in this investigation, it is essential to provide mechanical anchorage devices in order to guarantee a sufficient factor of safety, comparable to that generally accepted in conventional reinforced concrete design.

2. Up to the design load the load-deflection characteristics are practically identical regardless of whether or not the anchorage devices are retained, whereas above the design load, superior recovery properties are exhibited for the case where the anchorages are retained, even after appreciable cracking has occurred.

3. Where ample provision for bond is made, high strength concrete is essential to avoid premature failure by crushing.

4. Pre-stressing precluded the possibility of failure by diagonal tension and obviated the necessity of stirrups.

5. The required transmission length for 0.162" diameter steel wire is approximately 18 inches, or 108 times the diameter. This was not shown conclusively.

In general, it is felt that the investigation was successful, not only with respect to the actual tests themselves, but also from the point of view of the several techniques developed. Noteworthy is the method arrived at for determining the stress in wires by the tension-frequency relationship, and also the method of waterproofing the strain gauges.

It is realized that the foregoing conclusions have, of course, been drawn from the test of only two beams, and hence further investigations are required to provide adequate statistical evidence in support of the statements made.

Critique

During the course of the investigation, many imperfections came to light in the techniques employed and apparatus used that were not necessarily apparent at the outset. Accordingly then, means of improvement are now suggested in order that future investigators on the subject may benefit from the experience gained.

Perhaps the greatest single improvement that could be made lies in the manner of tensioning the wires. It will be recalled that the hydraulic jack used to apply the load to the steel wires, found reaction on the ends of the forms. On subsequent removal of the forms after the curing of the concrete, an 8% average loss of stress in the wires was recorded. This loss can be entirely eliminated by post-tensioning the wires.

If, however, it is desired to retain the pre-tensioning system, abutments could be provided between which the stressing of the wires could be effected without placing any load on the forms. By using either procedure the problem of having to provide a device for releasing load on the forms does not arise. Moreover, formwork could be constructed of cheap grade lumber, and cross sections of many varied shapes very easily formed.

It has already been noted that the device arrived at for releasing load on the forms in this investigation consisted of too many loose parts. If the same system is to be used again, a spot of welding to hold the angle struts to the channel web would greatly facilitate the initial assembly of the parts.

With reference to the gauge points, it is felt that more consistent readings would result if the housings were enclosed in metal tubes about 5/8" in diameter. The ends should be fitted with caps in which a centrally located hole is drilled to permit the passage of the steel wire.

Slots in the tube would be necessary through which the lead out wires could be brought. The whole tube should be free to slide longitudinally along the wire for about 1/4" without interference.

It should be possible to reduce the number of lead out wires to two at a gauge point. This could be done by connecting the two gauges in parallel.

During the actual testing of the beam it would be desireable to use Ames dials at all points where deflections are to be determined. The use of rulers does not permit a tolerable degree of precision.

Insofar as the actual design of the beam is concerned, it is now thought that the wires should be stressed higher (say to the order of 165,000 p.s.i.) and hence reduce the total number required. This would also reduce the number of anchors required as well as cut down the labor involved.

Suggested Topics for Further Investigations

1. It would be interesting to conduct a study on combining the advantages of pre-stressing with those of a continuous haunched girder, with an eye to arriving at the most economical proportions.

2. Various shaped I sections should be considered. Such beams could perhaps be constructed in the laboratory as a number of separate pre-cast sections held together by post-tensioned wires.

3. Attempts at measuring the strain on the outside of the concrete should be made, perhaps in the manner described in the Engineering News Record, March 8/51.

4. It is essential to know more about the creep of steel, both under constant load and under constant length.

5. Further investigation of bond stress is required. It is felt that the only approach here is through the use of very small (1/8") strain gauges which produce as little interference as possible with the continuity of the bond surface.

   Also on the subject of bond, it would be interesting to use twin twisted wires and conduct tests on their reliability without resorting to mechanical anchorages.

   Some thought should be given to an analytical method of computing bond stress. See the "Journal of the Institution of Civil Engineer" -- February, 1951 -- article by Prof. R.H. Evans.

6. Pending the development of some means of successfully measuring the stress in concrete, it would be of great value to determine the stress distribution over a section, sum integrally to get the total stress and compare this value with that computed from the bending moment.

7. Study should be made of the shear transfer characteristics between two precast units when held together by a tensioned cable.

8. Perhaps a short span could be built where shear becomes an important factor. The effectiveness of vertical pre-stressed reinforcing could then be studied.

APPENDIX "A"

APPENDIX "A"

Design of a Statically-Determinate Pre-Stressed Concrete Beam

The design procedure, outlined in the following pages, is the method presented by Gustave Magnel in his book "Prestressed Concrete". Fundamentally, the calculations involve the consideration of certain conditions which must be satisfied, but the number of variables is so great that a trial and error solution is the most expedient. Magnel, however, has shown how these basic relations may be illustrated graphically, from which the amount of pre-stress and eccentricity can be deduced once having determined the approximate dimensions of the concrete. The method can be applied to the design of elements of any shape.

Notation

The notation used in this chapter is the following:

A, cross-sectional area of beam.

b, breadth of rectangular beam or of flange of tee-beam.

C, C_1, constants relating to the ratio of stresses.

c, permissible compressive stress in the concrete.

c_{ab}, c_{at}, calculated stresses in the bottom and top fibres respectively due to w_a.

c_{db}, c_{dt}, calculated stresses in the bottom and top fibres, due to loads acting at the time of prestressing.

c_t, permissible tensile stress in the concrete.

D, total depth of beam or slab.

d, effective depth of beam or slab.

E_c, E_s, elastic modulus for concrete and steel respectively.

I, I_v, moments of inertia about the horizontal centroidal axis and about the vertical axis of symmetry respectively.

M, total bending moment.

M_a, M_a', bending moments due to w_a acting with and contrary to M_d respectively.

M_d, bending moment due to w_d.

P_i, initial stretching force.

Q, Q_1, factors for moment of resistance for ordinary reinforced concrete and for prestressed concrete respectively.

Q_m, moment about the neutral axis of the area of a section on one side of the neutral axis.

R, vertical component of force in an inclined cable.

r, radius of gyration of the concrete section.

S, shearing force. (S_{EA}, etc., shearing force at EA, etc.).

s, shearing stress

t, permissible tensile stress in the steel

w_a, additional load per unit length applied after the prestress has been established.

w_d, load per unit length acting when the prestress is being established.

y_1, y_2, distances from the centroid to the top and bottom fibres respectively; for a symmetrical section $y_1 = y_2 = y$.

e, eccentricity of the stretched wires from the centroid.

e_A, e_B, eccentricity applicable to points A and B respectively.

η, proportion of P_i that remains permanently; generally $\eta = 0.85$.

Fundamental Formulae

A prestressed concrete member subjected to bending only must in general resist a bending moment produced by the loads w_d present when the prestress is being established and to another bending moment produced by the loads w_a which can be applied after prestressing. Assume that the bending moments are such that compressive stress is induced in the top fibre of the concrete. The additional stress in the stretched wires due to the loads w_a will be neglected because this does not exceed 3 per cent. to 4 per cent. of the total stress in the wires, also because the stress induced in the wires during stretching decreases in time by an amount varying from 10 per cent. to 15 per cent. due to creep of the steel and concrete and shrinking of the concrete. At least 4 per cent. of the reduction in the stretching force occurs during the first few hours after the stretching is completed and is mainly due to the creep of the steel. Therefore, if the stress in the steel is not excessive at the moment the stretching is completed, it will never be so under the working load.

The section of the beam at mid-span must satisfy the following conditions.

Top fibre. (1) Immediately the prestress is established the tensile stress in the concrete under load w_d must not exceed the permissible tensile stress c_t in the concrete. This condition does not apply when the cable is within the core of the section, since the top fibre is then always in compression.

(2) After a time (say, one year), under loads w_d and w_a combined, the compressive stress in the concrete must not exceed the permissible compressive stress c in the concrete if the cable is below the core.

If the cable is within the core of the section, this condition must be fulfilled at the time the prestress is established. (In the foregoing the term "core" defines that zone of the beam, corresponding to the middle-third of a rectangular beam, within which the cable must be if tensile stresses in the concrete due to prestressing alone are to be avoided.)

Bottom fibre. (3) Immediately the prestress is established the compressive stress in the concrete under w_d must not exceed c .

(4) After a time, under the loads w_d and w_a combined, the tensile stress in the concrete must not exceed c_t .

These conditions can be expressed mathematically as in the following in which the symbols have the signification shown in Figure 1.

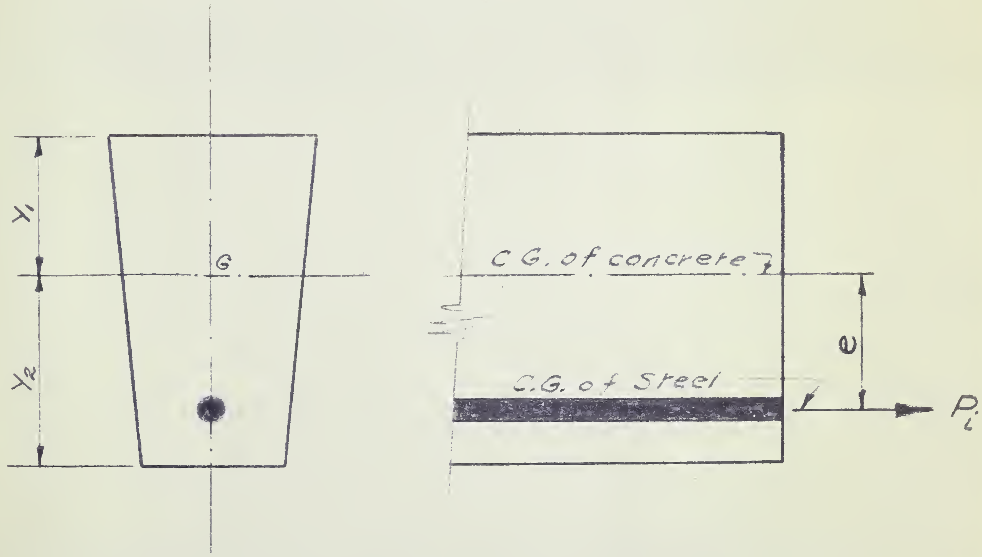


Fig. 1

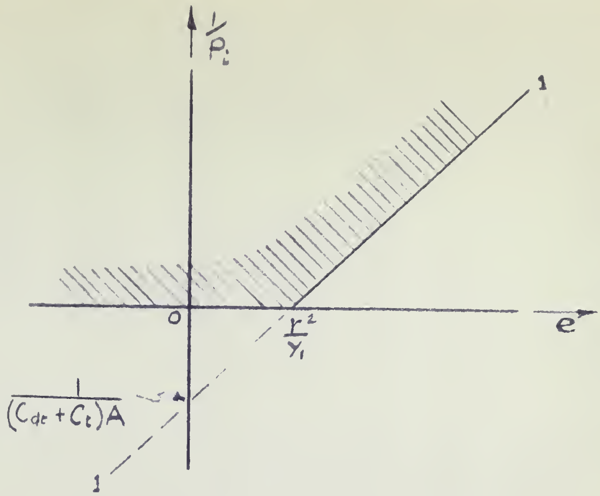


Fig. 2

Consider formula (2).

Case (i): $c \leq c_{dt} + c_{at}$.

If e is less than $\frac{r^2}{y_1}$ (Case 2), formula (2A) may be written

$$\frac{1}{P_i} \geqslant \frac{1 - \frac{e\, y_1}{r^2}}{(c - c_{dt} - c_{at})\, A} \quad \ldots\ldots\ldots\ldots \quad (14)$$

If e is equal to or greater than $\frac{r^2}{y_1}$ (Case 1), the condition in (2) is always satisfied for any value of P_i and e . It is clear that for a given section, the acceptable values of P_i and e (formula 14) are within the hatched area A in Figure 3.

This can best be done by changing slightly the form of the well known formula for fibre stress -

$$f = \frac{P}{A} \pm \frac{MY}{I}$$

or in terms of the symbols defined in the foregoing -

$$f = \frac{P_i}{A} \pm \frac{P_i\, e\, Y}{r^2\, A}$$

$$= \frac{P_i}{A}\left(1 \pm \frac{e\, Y}{r^2}\right)$$

Case (1) - e greater than $\frac{r^2}{Y_1}$

$$\frac{P_i}{A}\left(\frac{e\, Y_1}{r^2} - I\right) - c_{dt} \leqq c_t \quad \ldots\ldots\ldots\ldots (1)$$

$$-\eta\frac{P_i}{A}\left(\frac{e\, Y_1}{r^2} - I\right) + c_{dt} + c_{at} \leqq c \quad \ldots\ldots\ldots\ldots (2)$$

$$\frac{P_i}{A}\left(I + \frac{e\, Y_2}{r^2}\right) - c_{db} \leqq c \quad \ldots\ldots\ldots\ldots (3)$$

$$-\eta\frac{P_i}{A}\left(I + \frac{e\, Y_2}{r^2}\right) + c_{db} + c_{ab} \leqq c_t. \ldots\ldots\ldots\ldots (4)$$

Case (2) - e less than $\frac{r^2}{Y_1}$. Condition (1) cannot apply to the top fibre.

$$\frac{P_i}{A}\left(I - \frac{e\, Y_1}{r^2}\right) + c_{dt} + c_{at} \leqq c \quad \ldots\ldots\ldots\ldots (2A)$$

Formulae (3) and (4) remain as in Case (1)

With the exception of e, all the symbols used are numerical values; e is considered positive when measured downwards from G (Fig. 1), that is in the same direction as y_2; the contrary applies to continuous beams.

An important conclusion is derived as follows: Adding (3) and (4):

$$(I - \eta) \frac{P_i}{A} \left(I + \frac{e\, Y_2}{r^2} \right) + c_{ab} \leq c + c_t$$

Since $c_{ab} = \dfrac{M_a y_2}{I}$,

$$\frac{I}{y_2} \geq \frac{M_a}{c + c_t - (I - \eta) \dfrac{P_i}{A} \left(I + \dfrac{e\, y_2}{r^2} \right)} \quad \ldots\ldots\ldots (5)$$

In the same way, adding (1) and (2), if e is greater than $\dfrac{r^2}{y_1}$ (which is usual at sections at which the bending moment is greatest),

$$(I - \eta) \frac{P_i}{A} \left(\frac{e\, y_1}{r^2} - I \right) + c_{at} \leq c + c_t \ .$$

Hence

$$\frac{I}{y_1} \geq \frac{M_a}{c + c_t - (I - \eta) \dfrac{P_i}{A} \left(\dfrac{e\, y_1}{r^2} - I \right)} \quad \ldots\ldots\ldots (6)$$

If we assume that there is no loss of prestress ($\eta = I$) then, from (5) and (6)

$$\frac{I}{y_2} = \frac{I}{y_1} \geq \frac{M_a}{c + c_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

This indicates that the section must be sufficient to resist the bending moment due to the superimposed load w_a, the permissible stress being not c or c_t, but $c + c_t$. The load w_d (the dead load for example) existing when the prestress is being established does not influence the determination of the dimensions of the concrete. A prestressed concrete beam is thus smaller than an ordinary reinforced concrete beam.

Formula (7) satisfies only two of four conditions deduced from formulae (1) to (4); the other two are used to compute P_i and e, and it may be that the eccentricity thus determined may be too large for the beam of

the size computed from (7). Should this be so the depth of the beam must be increased. In cases where e is satisfactory, which is generally the case, the beam must be designed only for w_a, the load w_d, which is generally the dead load, being neglected. Thus an explanation is given of what is meant by saying that the weight w_d of a prestressed beam "carries itself". Even if there are losses of prestress, that is if η is less than 1, this statement is still nearly correct. In ordinary reinforced concrete the depth required at the middle of the span of a beam supported at both ends can be reduced by adding cantilevers at each end, thereby decreasing the total bending moment at the middle since the bending moment due to the dead load is decreased. In prestressed concrete, however, this is not the case, since the size of the beam at the middle is determined by considering only the bending moments due to loads that are imposed after prestressing, and these bending moments are not affected by the cantilevers, which might not be loaded.

First Calculation of the Dimensions

In most cases an upper limit to the terms $(I - \eta)\dfrac{P_i}{A}\left(I + \dfrac{e\,y_2}{r^2}\right)$ and $(I - \eta)\dfrac{P_i}{A}\left(\dfrac{e\,y_1}{r^2} - I\right)$ in formulae (5) and (6) can be determined fairly nearly, since η is generally about 0.85, $\dfrac{P_i}{A}$ is about 0.5c in a well-designed beam and $\dfrac{e\,y_1}{r^2}$ and $\dfrac{e\,y_2}{r^2}$ are each generally equal to about 2. It should be remembered that the evaluation of a secondary term can be done with a rather large error without appreciably affecting the final results. Hence the two terms referred to are approximately equal to 0.225c and 0.075c respectively, and formulae (5) and (6) now respectively become

$$\frac{I}{y_2} \geqslant \frac{M_a}{0.775c + c_t} \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

and

$$\frac{I}{y_1} \geqslant \frac{M_a}{0.925c + c_t} \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

With these formulae it is easy to determine the dimensions to adopt temporarily. For a symmetrical section $(y_1 = y_2)$, it is obvious that (8) should be used, since (9) is automatically complied with if (8) is satisfied.

Rectangular Beams

Formula (8) becomes

$$\frac{b\,D^2}{6} \geqslant \frac{M_a}{0.775c + c_t} \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

or

$$Q_1\, b\, D^2 \geqslant M_a \quad \ldots\ldots\ldots\ldots\ldots \quad (11)$$

where

$$Q_1 = \frac{0.775c + c_t}{6} \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

Graphical Representation of Formulae (1) to (4)

First consider formula (1). If e is equal or less than $\frac{r^2}{y_1}$ (Case 2), the condition expressed by (1) is always satisfied for any value of P_i or e. If e is greater than $\frac{r^2}{y_1}$ (Case 1), (1) may be written

$$\frac{I}{P_i} \geqslant \frac{\frac{e\,y_1}{r^2} - 1}{(c_{dt} + c_t)\,A} \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

It is clear that for a given section the acceptable values for P_i and e are within the hatched area of Fig. 2.

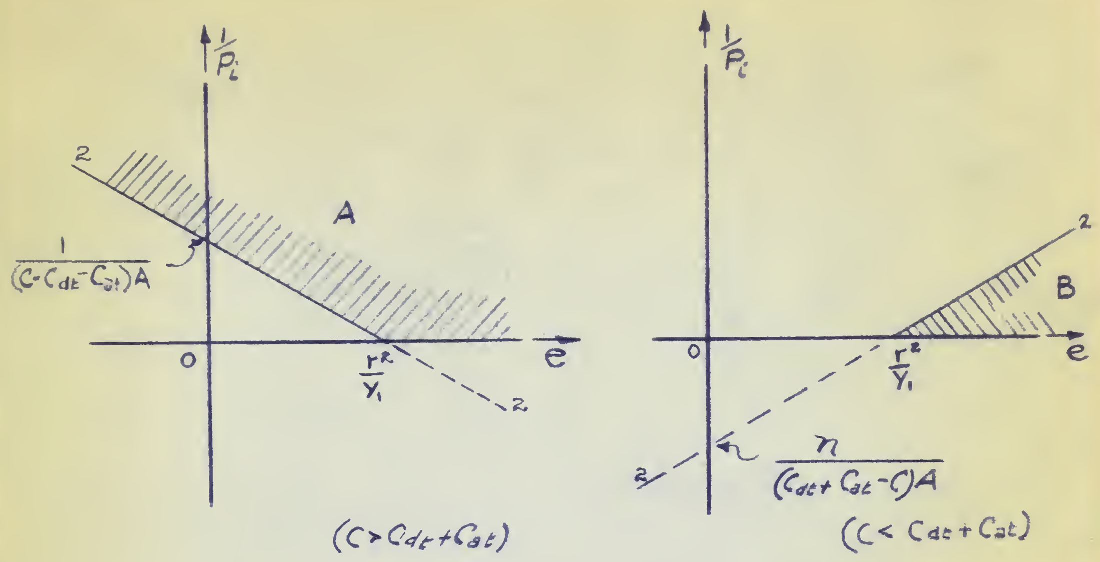


Fig. 3

Case (ii): $c \leq c_{dt} + c_{at}$.

If e is equal to or less than $\frac{r^2}{y_1}$ (case 2), the condition in (2) is never satisfied. If e is greater than $\frac{r^2}{y_1}$ (Case 1), formula (2) can be written

$$\frac{I}{P_i} \leq \frac{\eta\left(\frac{e\,y_1}{r^2} - 1\right)}{(c_{dt} + c_{at} - c)A} \quad \ldots\ldots\ldots\ldots (15)$$

Therefore for a given section the acceptable values of P_i and e are within the hatched area B in Fig. 3.

Consider formula (3). If e is equal to or less than $-\frac{r^2}{y_2}$, the condition in (3) is always satisfied. If e is greater than $-\frac{r^2}{y_2}$, (3) can be written

$$\frac{I}{P_i} \geqslant \frac{1 + \frac{e\,y_2}{r^2}}{(c + c_{db})A} \; \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

Thus for a given section the acceptable values of P_i and e are within the hatched area of Fig. 4.

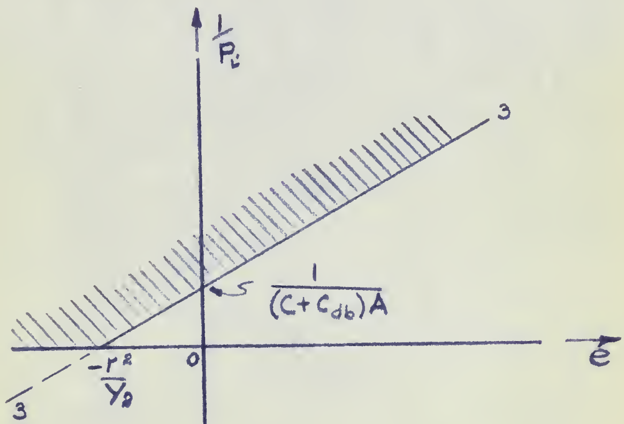


Fig.4

Finally, consider formula (4). If e is equal to or less than $-\frac{r^2}{y_2}$, the condition in (4) is never satisfied in practice. If e is greater than $-\frac{r^2}{y_2}$, (4) may be written

$$\frac{I}{P_i} \leq \frac{\eta\left(1 + \frac{e\, y_2}{r^2}\right)}{(c_{db} + c_{ab} - c_t)A} \quad \ldots\ldots\ldots\ldots \quad (17)$$

Therefore for a given section the acceptable values of P_i and e are within the hatched area of Fig. 5.

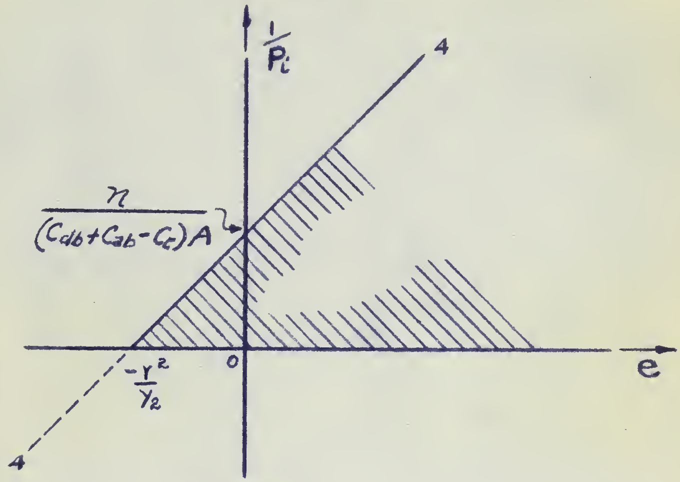
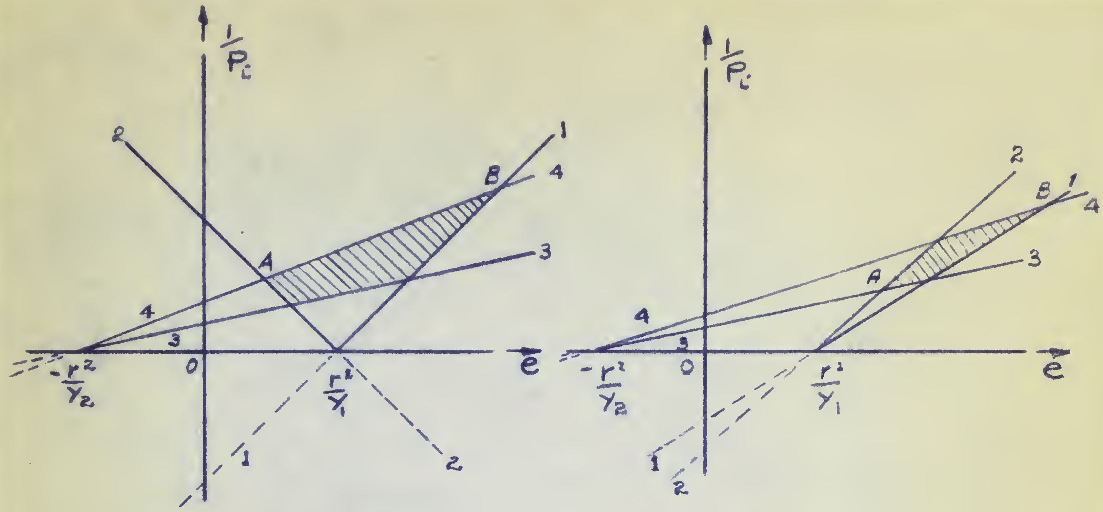


Fig.5

The four diagrams in Figs. 2 to 5 are combined in Fig. 6(a), which applies if c is equal to or greater than $c_{dt} + c_{at}$. The acceptable values of P_i and e that satisfy at the same time the conditions in the four formulae are within the hatched area of Fig. 6(a). That is, if e is greater than 0, line 3 must be below line 4, otherwise there is no solution in which P_i and e satisfy at the same time the conditions in the four formulae.



Case: $-C \geq C_{dt} + C_{at}$
(a)

Case: $-C \leq C_{dt} + C_{at}$
(b)

Fig. 6

If c is less than $c_{dt} + c_{at}$, then Fig. 6(b) results. The acceptable values of P_i and e satisfying at the same time the conditions in the four formulae are within the hatched area. That is, if e is greater than $\frac{r^2}{y_1}$, line 3 must be below line 4 and line 1 below line 2, otherwise there is no solution in which P_i and e satisfy at the same time the conditions in the four formulae.

Limiting Eccentricity

Let us compute the abscissa e_A of the point A (Fig. 6(a)), the intersection of lines 2 and 4. If e is less than $\frac{r^2}{y_1}$, the equations for these lines are

$$\frac{P_i}{A}\left(1 - \frac{e\,y_1}{r^2}\right) = c - c_{dt} - c_{at} \quad \text{and} \quad \frac{\eta P_i}{A}\left(1 + \frac{e\,y_2}{r^2}\right) = c_{db} + c_{ab} - c_t \,.$$

By dividing the first by the second of these formulae and substituting

$$C = \eta \frac{c - c_{dt} - c_{at}}{c_{db} + c_{ab} - c_t} \quad \ldots\ldots\ldots\ldots\ldots \quad (18)$$

the formula for e_A is

$$e_A = \frac{(1 - C)\, r^2}{y_1 + C\, y_2} \quad \ldots\ldots\ldots\ldots\ldots \quad (19)$$

In the same way, compute the abscissa e_A of point A (Fig.6(b)), the intersection of lines 2 and 3. The equations for these lines are

$$\frac{\eta P_i}{A}\left(\frac{e\,y_1}{r^2} - 1\right) = c_{dt} + c_{at} - c \quad \text{and} \quad \frac{P_i}{A}\left(1 + \frac{e\,y_2}{r^2}\right) = c_{db} + c \,.$$

By division and substituting

$$C_1 = \frac{c_{dt} + c_{at} - c}{\eta(c_{db} + c)} \quad \ldots\ldots\ldots\ldots\ldots \quad (20)$$

$$e_A = \frac{(C_1 + 1)\, r^2}{y_1 - C_1 y_2} \quad \ldots\ldots\ldots\ldots\ldots \quad (21)$$

If the greatest eccentricity obtainable with the wires within the chosen section is less than e_A obtained from (19) or (21), then no combination of P_i and e will satisfy at the same time the conditions in the four formulae.

Practical Calculations

A section satisfying formulae (8) and (9) is chosen. The stresses c_{dt}, c_{at}, c_{db} and c_{ab} are then computed. Next e_A is computed from (19) if c is greater than $c_{dt} + c_{at}$, or from (21) if c is less than $c_{dt} + c_{at}$. If the greatest eccentricity obtainable with the wires within the chosen section is less than e_A, then the depth of the section must be increased. If the greatest eccentricity obtainable is greater than e_A, a diagram such as that in Figs. 6(a) and (b) is drawn.

If the greatest eccentricity obtainable exceeds e_B, the eccentricity corresponding to point B in either Figs. 6(a) or (b), e_B is adopted with the corresponding initial prestress indicated by the diagram; that is, P_i is determined from the ordinate of B. If the greatest eccentricity obtainable is less than e_B in either Figs. 6(a) or (b), the greatest eccentricity available is adopted, and the value of P_i is determined from the ordinate to the point on the upper boundary of the hatched area, the abscissa of the point being the value of the greatest eccentricity obtainable. (Note that the greatest value of $\frac{1}{P_i}$ corresponds to the smallest value of P_i). The examples given later make this method clear.

Effect of Dead Load at the Time the Prestress is Established

To simplify the explanation of the statement previously made that "the dead load carries itself", the loss of prestress will be neglected (that is $\eta = 1$), and a symmetrical section $(y_1 = y_2 = y)$ will be assumed. Formula (7), which results from combining (1) + (2) and (3) + (4), having been satisfied, (1) and (3) remain to be complied with in order that the four fundamental conditions, now equalities, be fulfilled. Combining (1) and (3) for a symmetrical section, and with $c_{db} = c_{dt}$

$$\frac{\dfrac{e\,y_1}{r^2} - 1}{1 + \dfrac{e\,y_1}{r^2}} = \frac{c_t + c_{dt}}{c + c_{dt}}$$

Hence $$e = \frac{c + c_t + 2c_{dt}}{c - c_t} \cdot \frac{r^2}{y_1}$$

If tensile stress is inadmissible $(c_t = 0)$,

$e = \left(1 + \dfrac{2c_{dt}}{c}\right)\dfrac{r^2}{y_1}$. With the value of c_{dt} obtained from $\dfrac{M_d\,y}{I}$ (assuming that $c_{dt} = c_{db}$) and combining with formula (8) written as an equation with $y = y_1$,

$$e = \left(1 + 1.55\,\frac{M_d}{M_a}\right)\frac{r^2}{y} \quad \text{. . . . . (21A)}$$

With this value of e , equation (1) becomes

$$P_i = \frac{M_a}{1.55\,\dfrac{r^2}{y}} \quad \text{. . . . . (21B)}$$

In brief, (1) to (4) have been replaced by (7), (21A), and (21B). In the case considered, the superimposed load alone influences the dimensions of the concrete and the value of P_i , while the eccentricity alone depends on the loads acting when prestressing is completed. It must not be concluded from this that for a beam, which has to carry only the load present when prestressing is completed and which may be not only the dead load, $\frac{I}{y} = 0$ and $P_i = 0$, according to (7) and (21B), so long as e has a sufficiently large value; as a matter of fact e would be infinity according to (21A). Although theoretically correct, it is impossible in practice. If w_a is small in relation to w_d , e becomes larger than is acceptable.

For a rectangular section, a slab for example, $\frac{r^2}{y} = \frac{D}{6}$; but e can never exceed, say, $0.9 \frac{D}{2}$, that is, $0.45\ D$. According to (21A) these conditions correspond to $\frac{M_d}{M_a} = 1.10$. Therefore it is obvious that when M_a is less than $0.91\ M_d$ the dimensions given by (7) lead, through (21A), to an eccentricity which cannot be realised in practice unless the cables are placed outside the concrete, in which case (7) could still give a solution for values of M_a smaller than $0.91\ M_d$; but, again, there is a limit to the distance from the slab at which the cables can be placed.

It is emphasised that (21A) and (21B) must not be used by themselves; they are inseparable from (7). In other words, if (7) is departed from, (21A) and (21B) have no value. Formulae (21A) and (21B) are not given for use in practical calculations but only to explain plainly what is meant by the statement "the dead load (that is the load existing at the moment the prestress is established) carries itself".

APPENDIX "B"

APPENDIX "B"

The Frequency Method for Checking Stress in Wires

One of the problems encountered in the course of the investigation was that of determining the stress in the wires after they had been tensioned.

By tapping the fluid reservoir of the jack used to apply the tensioning load and introducing a hydraulic gauge into the line, a reasonable value of the applied load may be obtained. However, even if the gauge is reliable, the load thus obtained is not the true load to which the wires are subjected after the jack has been removed. This follows, since in the transfer of the load from the jack to the gripping devices a considerable relaxation of load is permitted.

Another solution is the installation of wire-resistance strain gauges on every wire, or perhaps on one of every pair of wires which are to be tensioned simultaneously. However, this entails a good deal of preparation and is also a very expensive process.

The method finally adopted made use of the tension-frequency relationship for a vibrating uniform string. The theory for arriving at this relationship is now given.

Consider a string, vibrating between two fixed points A and B as in Figure 1 (a). The equation of motion is derived by writing Newtons law for a small element dx of the string. The tension T is assumed to be constant.

The deflection curve during the vibration will be represented by some function $y(x, t)$. That is, the ordinate varies with both the location along the string and with time.

"B"

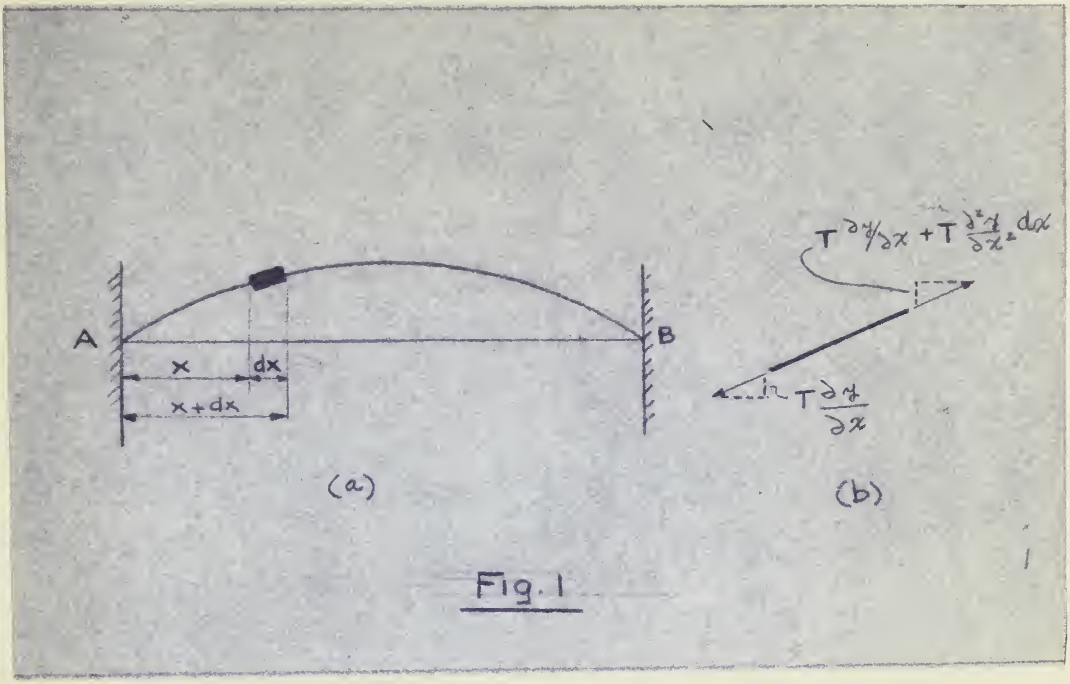


Fig. 1

The vertical component of the tension T pulling to the left at a certain point x of the string is (Figure 1(b))

$$- T \frac{\partial y}{\partial x}$$

negative because it acts downward whereas y is positive upwards. The partial derivative is taken because the string is considered at a certain instant, i.e. t is constant in the differentiation.

At the right hand end of the element dx the vertical component of the tension is

$$T \frac{\partial y}{\partial x} + \partial \left(T \frac{\partial y}{\partial x}\right) = T \frac{\partial y}{\partial x} + \frac{\partial}{\partial x} \left(T \frac{\partial y}{\partial x}\right) dx$$

$$= T \frac{\partial y}{\partial x} + T \frac{\partial^2 y}{\partial x^2} dx$$

This quantity is positive because it acts upwards. $\frac{\partial^2 y}{\partial x^2} dx$ expresses the increase in slope along dx.

By Figure 1b it is noticed that there is an excess upward pull of $T \frac{\partial^2 y}{\partial x^2} dx$ which must accelerate the element in an upward direction.

If μ_1 = mass per unit length of the string then Newtons law gives

$$\mu_1 \, dx \frac{\partial^2 y}{\partial t^2} = T \frac{\partial^2 y}{\partial x^2} dx$$

$$\therefore \quad \mu_1 \frac{\partial^2 y}{\partial t^2} = T \frac{\partial^2 y}{\partial x^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

which is the differential equation of motion for a vibrating uniform string.

Assuming that the string vibrates harmonically at some natural frequency, the function $y(x,t)$ is of the form

$$y(x,t) = f(x) \sin \omega t$$

Substituting in Eqn.(1)

$$\mu_1(-\omega^2 f(x) \sin \omega t) = T \frac{d^2 y}{dx^2} \sin \omega t$$

from which

$$\frac{d^2 y}{dx^2} + \frac{\mu_1 \omega^2}{T} y = 0$$

The general solution to this differential equation is of the form

$$y(x) = C_1 \sin x \sqrt{\frac{\mu_1 \omega^2}{T}} + C_2 \cos x \sqrt{\frac{\mu_1 \omega^2}{T}}$$

From this it can be shown that

$$T = 4 \ell^2 \omega^2 \frac{W}{g}$$

where W = wt. per foot of the string

g = acceleration of gravity in ft/sec^2

ℓ = length in ft. of freely vibrating string

T = tension in #.

ω = frequency of vibration in cycles/sec.

Thus, if the frequency of vibration is known, the remaining factors given, then the computation of the tension is a simple matter.

The highly desirable feature of this method is that it permits the determination of the actual tension in the wire as it rests in its final position stretched between the form ends. Furthermore the tension can be checked at any time up until the actual placing of the concrete commences.

The apparatus used was relatively simple as the photograph Figure 2 will reveal.

It consists of:

(a) a magnetic pick-up

(b) an audio oscillator

(c) an oscilloscope

(d) an amplifier

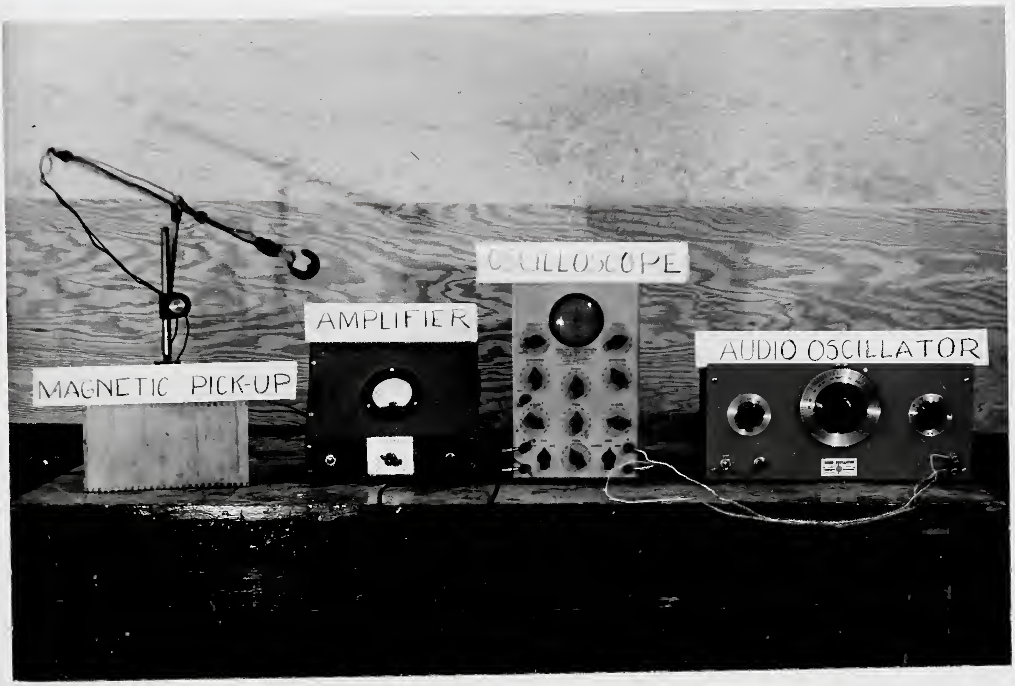


Fig. 2

The amplifier may or may not be necessary depending upon local conditions of interference and the type of accompanying apparatus. The magnetic pick-up used in this investigation was constructed from the permanent magnet and windings of a discarded earphone.

A schematic diagram is given (PLATE XVIII) for the hook-up used.

The operation of the equipment is as follows: the magnetic pick-up is placed as close as possible to, but without at any instant touching, the wire whose tension it is desired to know. The wire is then dealt a sharp blow setting it into vibration. The audio oscillator is then adjusted until the screen of the oscilloscope indicates a slowly revolving single trace circle (or ellipse). Finer adjustment will result in complete stoppage of the revolutionary motion. At this point, then, the frequency may be read from the dial of the audio oscillator.

This frequency is equal to the frequency of vibration of the wire under study. A pre-computation of the frequency facilitates finding the approximate setting on the oscillator initially.

Some interference was at first experienced from 60 cycle power generating installations located very near by. This problem was overcome by introducing a special choke circuit into the amplifier.

It is the author's opinion that this apparatus could be refined and consolidated into one self-contained unit which would be extremely suitable for commercial adaptation. In this regard it is suggested that some bridging device be designed to permit the frequency determination to be made constantly over some one convenient length. With this quantity fixed, and the range of desired stresses known, a special audio oscillator (or other device) could be designed to operate over a small range of frequencies, but calibrated to permit greater precision in reading, than is possible with normal laboratory equipment.

It should be noted, of course, that this method of checking stress is limited to the pre-tensioning type of systems.

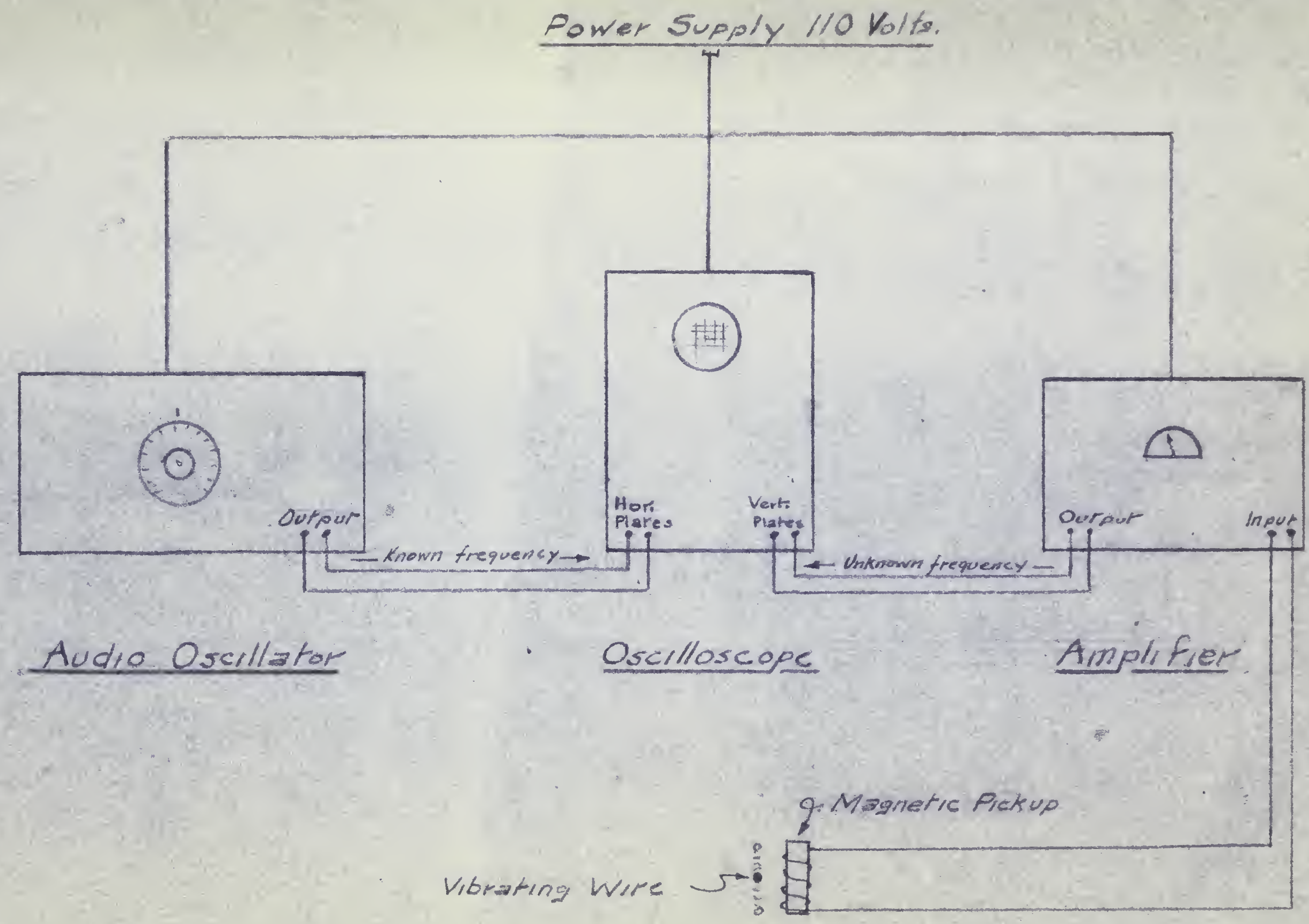


PLATE XVIII

FREQUENCY DETERMINATION

Electrical Hook-up

APPENDIX "C"

APPENDIX "C"

Method of Attaching & Waterproofing Wire-Resistance Strain Gauges

A. Application of gauges to small diameter reinforcing steel

1. Using fine grade emery cloth, clean the surface of the steel wire in the area where the gauge is to be attached.
2. Ensure cleanliness of this area by polishing with a piece of white cloth saturated in acetone. Continue this until no smudge is left on a clean piece of cloth.
3. Apply Duco Household Cement (or Dupont No. 5458) to the back of a gauge from which the excess paper has been trimmed from the sides. Do not wait for this cement to become tacky as per some instructions.
4. Apply the gauge to the steel wire and gently but firmly bind with a heavy grade cotton thread. This must be very carefully done in order to avoid damaging the fine wires within the gauge. At the completion of this step, and while the cement is still wet, check the alignment of the gauge. It should, of course, have its long axis in line with the length of the wire.

Fig. A shows a typical gauge point at this stage of the procedure. Note that two gauges are required for a single gauge point, the gauges being placed end to end and on opposite sides of the wire. This is to ensure determination of a correct value of stress even if the wire is not straight at the time of taking the "zero" readings on the gauges.

B. Attaching lead out wires

1. Solder lead out wires to the fine leads of the gauges. It is essential that the lead out wires have a waterproof insulation. G.E. Flamenol No. 18 has been used successfully for this.
2. Using cotton thread, bind the soldered joint to the steel wire making sure that all uninsulated wire bears on the felt covering of the gauge. Coat liberally with Duco cement.
3. At a point about 1" away from the end of the gauge, anchor the lead outs to the steel by means of a wire binding. This should be firm enough to take any pull that may come on the leads. This is extremely important, for any movement transmitted to the gauges in this manner can render extremely inaccurate readings.
4. Trim off excess ends of thread and wire. See Fig. B for this step.

C. Waterproofing the gauges

1. Using a $\frac{1}{2}$" paint brush, apply an even coat of molten "Petrolastic" asphalt to the gauges, the surrounding steel wire, and lead out wires.
2. Repeat step 1 taking care to fill any holes in the first coat. A third coat may even be necessary. In this regard, it is better to use several thin layers of asphalt rather than one thick coat. This procedure permits the heat to dissipate more rapidly into the air, rather than "going in" to the gauge. If the latter occurs there is a danger of the cement being scorched to the extent that the gauge becomes loosened from the steel.
3. If the coating has become overly bulky in some areas, a heated soldering iron is useful for smoothing this out. This method also works well for "feathering" out and sealing the edges.

Fig. C illustrates the appearance at this point.

C. <u>Waterproofing the gauges</u> (contd)

4. Finally paint on a sealing coat of pigmented aircraft dope. The action of the dope is twofold.

   (a) Its fluidity makes it easy to apply and thus ensures a perfect seal.

   (b) It has a tendency to go into solution with the other layers of asphalt and form a pliable substance that does not crack when the lead out wires are handled.

   A completed gauge point is shown in Fig. D.

Fig. A Gauges bound & glued to steel.

Fig. B Lead out wires secured in place.

Fig. C Ashphalt housing completed.

Fig. D Seal coated with aircraft dope.

APPENDIX "D"

APPENDIX "D"

Design of the Forms

The forms used in the construction of the beams were designed to the following requisites.

1. The forms were to be capable of withstanding the pre-stressing load without appreciable deformation, since a pre-tensioning system was to be used to stress the wires where the forms were to provide reaction to the loading jack.

2. Some device was required to release the load on the forms to permit their removal from the concrete at the end of the curing period.

3. Provision was to be made to enable beams of varying depth-width ratios to be constructed.

4. They were to be durable enough to withstand continued reuse in future investigations.

With the above points in mind it was decided that the forms should be constructed from structural steel. This choice is almost dictated by requirement (1) above. In the course of an earlier investigation of pre-stressed concrete at this University, wooden forms were used. These proved unsatisfactory in that an appreciable percentage of the initial prestress was lost due to the crushing of the wood. Moreover, this crushing effect was not uniform and some of the wires were materially more affected than others. The reuse of these forms was also not a feasible proposition.

For the main members (i.e. the form sides) 2 - 12" channels @ 20.7# were selected. The bottom plates were cut from 1/8" sheet steel with holes provided to permit their bolting to the bottom flanges of the

channels. A pattern of holes was designed for the plates such that the channel spacing could be changed to permit the construction of beams varying in width from 6 to 12 inches inclusive in increments of whole inches.

Five pieces of 1/4" strap were provided to tie the top flanges of the channels together.

The end plates were specified as 3/4" stock. Even with material of this thickness, a considerable loss of pre-stress was attributable to the deformation of the plate under load. However it must be remembered that a good deal of man-handling of this plate is necessary and hence material sufficiently thick to resist appreciable deformation becomes impractical. Perhaps a short length of WF section could be utilized to better advantage A copy of the detailed drawings used in ordering the forms may be found at the end of the report.

The provision of a device for releasing the load placed on the forms during the tensioning operation, gave some difficulty. Many schemes were arrived at and subsequently discarded because of their complexity and/or cost of fabrication. The system finally adopted worked satisfactorily insofar as the releasing of the load was concerned. In the light of practical usage, it is now criticized as being comprised of too many loose parts. This is particularly annoying during the initial assembly stages before any load is present to hold the parts in place. Suggestions for improvement have been set forth under a general critique covering the entire investigation.

The device consisted of

2 stop plates - 10" x 9" x 1/2"

4 L's - 3" x 2" x 1/2" x 9 1/8"

Reference is made at this point to the Jacking End Detail of the main blueprint for an assembly view of the device.

The stop plates were welded to the outside of the channels such that one edge was exactly 9" from, and parallel to, the end of the channel. The pieces of angle 9 1/8" in length acted as struts between the stop plates and the end plates. Thus a gap of 1/8" existed at all points between the ends of the channel and the end plates. This was sealed off inside to prevent the concrete mortar from filling the gap.

The leg of the channel adjacent to the web of the channel was tapped at two places to receive two 5/8" diameter bolts.

When it was desired to release the load on the forms, these bolts were tightened. As their ends made contact with the channel web, the angles were gradually forced outwards and away from the forms until they were clear of the stop plates and hence fell away. All bearing surfaces were machine finished and well greased. By using a suitable socket wrench no difficulty was experienced in tightening the bolts.

Because of the 1/8" gap between the channels and the end plates, reaction to the load must under these conditions be provided by the end of the concrete beam. The forms thus relieved of their load were removed with comparatively little difficulty. The forms appear in several of the photographs of the section "Test Procedure".

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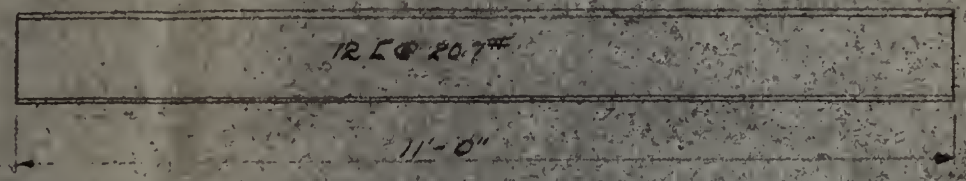
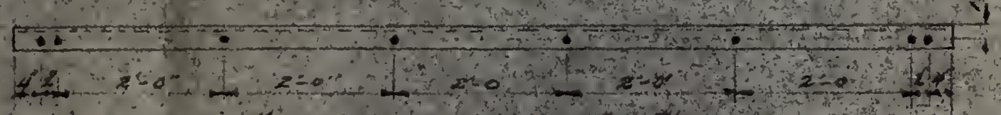
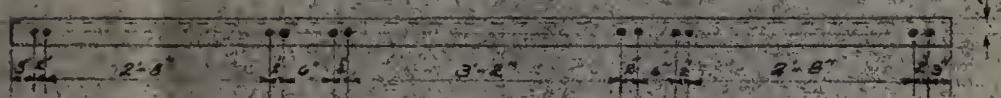
Print of Steel Forms.



Elevation of Form Side

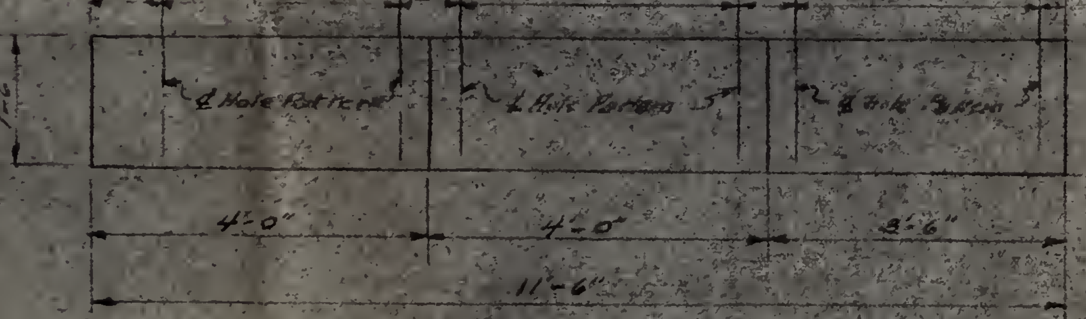


View showing Holes - Top Flange

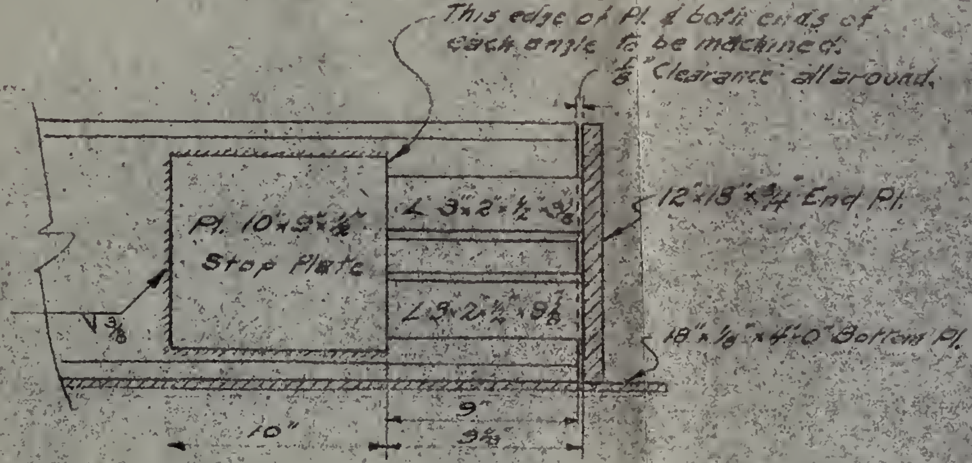


View showing Holes - Bottom Flange

Note - All holes 9/16" Dia.



Bottom Plate

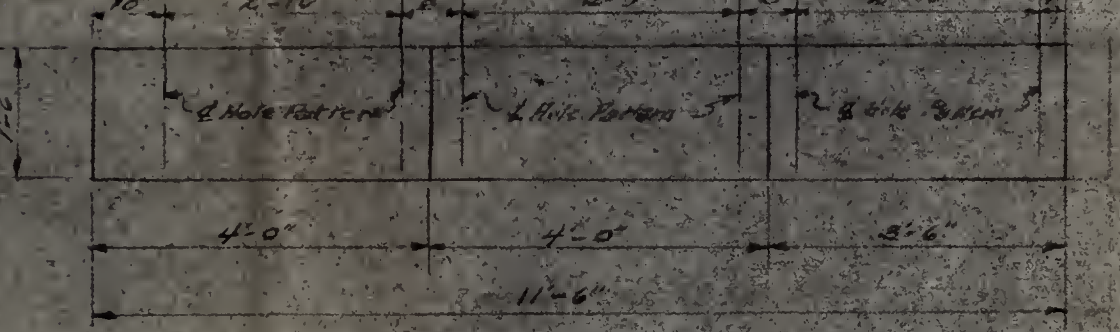


Jacking End Detail
(1½" = 1'-0")

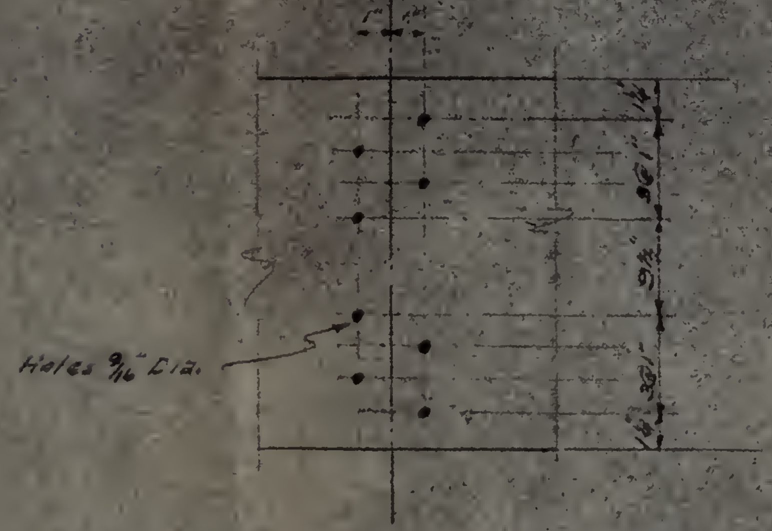
Plain End Detail
(1½" = 1'-0")

BILL OF MATERIALS

Plain End Detail
(1½" = 1'-0")



Bottom Plate



Detail showing Hole Pattern for Bottom Plate
(Not to scale)

BILL OF MATERIALS

Item	No.	Dimensions	Weight
Form Sides	2	12" - ['s @ 20.7 - 11'-6"	455.4#
Bottom Pls.	2	18" x 1/8" x 4'-0"	61.2#
" "	1	18" x 1/8" x 3'-6"	26.8#
End Pls.	2	12" x 18" x 3/4"	84.2#
Braces	4	L's - 3" x 2" x ½" - 0'-9⅛"	25.1#
Stop Pls.	2	10" x 9" x ½"	12.8#

Note:- Above list is for one form only, 2 req'd.

UNIVERSITY OF ALBERTA
Dept. of Civil Engineering

PRESTRESSED-CONCRETE
BEAM FORMS

Scale ½" = 1'-0" (except as noted)
Date: Nov. 1950